New Approaches to the Study of the Measurement Problem in Quantum Mechanics

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The most difficult problem, however, concerning the use of the language arises in quantum theory. Here we have at first no simple guide for correlating the mathematical symbols with concepts of ordinary language; and the only thing we know from the start is the fact that our common concepts cannot be applied to the structure of the atom.

Werner Heisenberg [Hei59, page 153]

Whereof one cannot speak, thereof one must be silent.

Ludwig Wittgenstein [Wit22, §7]

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Dissertation Summary

The dynamics and the postulate of collapse are flatly in contradiction with one another ... the postulate of collapse seems to be right *about what happens when we make measurements, and the dynamics seems to be bizarrely* wrong *about what happens when we make measurements, and yet the dynamics seems to be* right *about what happens whenever we* aren't *making measurements; and so the whole thing is very confusing; and the problem of what to do about all this has come to be called "the problem of measurement"*

David Albert [Alb92, page 79]

The problem of measurement lies at the heart of the theory of quantum mechanics. It disturbed the founders of the theory, as well as other great thinkers throughout the century since then. Bohr had to resort to intricate intellectual exercises in order to explain the effect of the macroscopic measurement apparatus on the microscopic quantum system (see section 1.1.6). Schrödinger was quoted saying: "If all this damned quantum jumping were really to stay, I should be sorry I ever got involved with quantum theory" [Jam74, page 57]. He then conceived of the cat story [Sch35a] in order to express the difficulties posed by the quantum measurement. Wigner had to resort to a friend in a sealed room [Wig63]. Von Neumann had to add a special process in order to account for the non-unitary development following quantum measurement (see section 1.1.3).

In this dissertation, I will try to give some twists to the problem of measurement. The main idea is to use subtle kinds of measurement, ones that do not trigger a complete reduction of the quantum state. Such delicate measurements bring to light some interesting implications of the process of measurement, and it is hoped that they will help us to decipher some deeper ideas, and maybe get some better understanding of the process.

By presenting thought experiments I will survey the following issues:

- The non-local effects exerted during measurement of entangled quantum system.
- The relations between quantum measurement and erasure.
- Non-sequential interactions that seem to ghostly pass without interaction through several measurement devices, then affect others, and then skip some others again.
- Measurements that seem to affect the entire history of the development of a quantum system.
- An apparent paradox where two particles use entanglement in order to "deny" the very fact that they are entangled.

These results suggest a strong relation between the quantum world and the problem of measurement on the one hand, and the notions of space and time on the other hand. These relations will be surveyed at the dissertation summary.

The Purpose of This Research

In the intellectual history of mankind, there has never been a challenge as difficult as comprehending of the conceptual foundations of quantum mechanics. On one hand, the mathematical formulation of the theory enables us to forecast experimental results with an unprecedented accuracy, but on the other hand, its fundamental concepts defy the most basic intuitions known to us from our everyday experience in the macroscopic world. Notions like "object," "particle," "position," "trajectory" and even "time" and "causality" lose their known meanings in favor of new and surprising meanings and context.

One of the central problems one runs into when trying to understand quantum mechanics is the measurement problem. The measurement process connects the microscopic quantum system with the macroscopic device, thereby connecting the conflicting quantum and classical notions. The observed macroscopic result does not accord with the unitary evolution of quantum systems, which perfectly hold at the microscopic realm. This problem was discussed in the literature ever since the first days of quantum mechanics, and lies at the basis of our understanding of the quantum theory [Jam66, page 349].

The measurement process itself is a multi-stage process, which begins with the initial interactions at the quantum level, continues with amplification using complex, nonlinear, processes, and ending at the macroscopic level, where the quantities of particles are of the order of magnitude of Avogadro number ($\approx 10^{27}$). In spite of this complexity, it is customary to reduce the measurement process into a single interaction taking place during a short time, according to **John von Neumann**'s Measurement Theory [vN55, chapter VI]. This oversimplification, as I will try to show in this work, blurred the scientific and philosophical difficulties involved in the measurement concept.

A deeper problem troubling physicists and philosophers is the meaning of the fundamental notions of quantum mechanics. Physicists are used to talk about "quantum particles", but evidently, those particles do not behave like the particles we know from our everyday, macroscopic experience. In addition, the quantum formalism has no notion of "particle[1]"; rather, it offers a "wave function" or "state vector," which gives a merely operational description of the probabilities for the possible measurement results. What, then, can we say about these fundamental entities of the physical world? Are they particles or waves? Or maybe these fundamental entities do not comply with any of these categories of the macroscopic world? Or maybe reality has many faces?

Above all these riddles hovers the metaphysical question: Does quantum mechanics provide an ontology, that is, does it describe reality itself, or merely epistemology, namely, our (limited) knowledge about reality?

In this work, I will try to illuminate the measurement problem, and the philosophical questions involved, from a different angle. The methods I will apply will be based on the idea of regarding the measurement process as a continuous and complex one. I will also make a wide use of "gedanken" or "thought" experiments. Gedanken experiments were already used by the pre-Socratic philosophers in ancient Greece, and later by Galileo, Newton and Einstein, through the present. These experiments, done solely in imagination (although, if presenting interesting results, are often carried out in reality as well), allow the scientist and the philosopher to construct complex experimental settings, and to use them to evaluate the logical basis of a theory. This way, free from the limitations of their contemporary technology, they can test the theory in the most extreme situations, exposing its fundamental weaknesses.

[1]Notwithstanding Bohm's interpretation, as I will elaborate later on. However, even in Bohm's interpretation, the "corpuscles" are distinctly different from the entities that we usually associate with the notion "particle", see, for example [BDH95].

Many important experimental results came into being in that way.

Specifically, I intend to use thought experiments in order to investigate complex quantum interactions which constitute some kind of measurement, yet are different from the ordinary quantum mechanical measurement, as formulated by von Neumann. One kind of such a measurement is the Partial Measurement, where the measurement is performed on only part of the wave function. Such a partial measurement can occur when the measurement is performed only on a section of the region in space the wave function is spread over, or when the particle is in a state of superposition and the measurement is designed to measure only a subset of the superposed values.

When such a measurement ends up with no particle measured (negative, or null result), the wave function undergoes a certain change, one that is experimentally measurable. It is possible to increase and decrease the part of the wave function undergoing measurement, and thereby continuously shift from no measurement to a full measurement.

Another kind of special measurement is one where the measurement apparatus itself is a quantum system, prepared in a state of superposition. Such a system allows for extremely delicate measurement and displays extraordinary results in the measured system, allowing us better understanding of quantum behavior.

The Plan of This Research

This work is divided into three parts. In the first one I will elaborate the required theoretical background (Chapter 1), including a historical review of the basic concepts to be used later, while expounding on the fundamental problems emanating from these concepts. Then I will elaborate on the theoretical tools that will be used later within the thought experiments: Interaction Free Measurement (Chapter 2) and Quantum Erasure (Chapter 3).

In the following part, Chapters 4-6, I will describe complex gedanken experiments using the tools introduced previously. In these chapters I will explore the various stages of complex interactions. Analyzing such interactions requires long, elaborate and precise arithmetical calculations, even for simple interactions, involving a couple of particles. Until recently, such analysis would have required long and frustrating manual calculation. Today, thanks to algebraic analysis software, such a research is conducted quickly and easily, freeing the researcher from the limitations of the calculation and allowing him or her to concentrate on fundamental issues. For the purpose of this work I use the MatLab software for the algebraic analysis.

In the last part (Chapter 8), I will analyze the results from various viewpoints: first, I will evaluate the implications with regard to our notions of "particle" and "wave", or the realism of the quantum state.

In addition to the question of realism, these results often highlight problems in our concepts of time and causality. At times, it appears as if quantum measurements "determine" the state of the system retroactively, hence it is mandatory to carefully investigate the causal chain of events.

Part I

Review of the Relevant Works since the Early Days of Quantum Mechanics to the Present

Chapter 1

Historical Background

Since this thesis is addressed to both physicists and philosophers, I have to set a common ground for both. In this chapter I will describes both the required physical background for the philosopher, and the basic philosophical ideas to be used hereafter.

However, I will not discuss the various interpretations of the quantum theory – a task that lies beyond the scope of this research. A thorough discussion of that topic can be found in [Jam74, chapters 7-10], [Hug89, part II] or [Hom97, chapter 2].

1.1 The Main Problems of Quantum Mechanics

Since the end of the 17^{th} century, Western civilization's scientific world-view became sharper and more precise, focusing on the following fundamental ideas:

1. There is real existence to the universe, regardless of human beings or any other observers watching it.

2. Some of the existing entities are not amenable for direct observation. Their existence is revealed

only through the physical theories. Amongst these entities one can enumerate the absolute space, absolute time, matter particles and the various forces and fields.

3. Using our wit, it is possible for us to decipher the secrets of the Universe and reveal the fundamental entities and their rules of action.

4. Using measurements with high enough precision, it is possible to predict or retrodict the future or past states of every physical system.[1]

These concepts were assimilated deeply into Western culture – or perhaps conversely, Western culture was crystallized around them. Either way, they constitute basic intuitions of nearly every Western person.

On the advent of the 20^{th} century, this monolithic picture began cracking: The Special Theory of Relativity taught us that space and time are not absolute but depend on the observer's frame of reference; that matter and energy are interchangeable; and that even mass and energy themselves are not absolute but relative to the frame of reference [Ein35b]. At the same time (notwithstanding special relativity's disapproval of concurrency...), quantum mechanics showed that there is a theoretical limit to our ability to observe reality: for certain systems, one can predict no more then the probabilities for the results of various experiments. Moreover, the theoretical description of Nature called for a picture remotely detached from our everyday experience: many physical properties come in fundamental discrete quantities, all physical entities are fundamentally described as waves, and these entities can retain simultaneous multiple values for physical properties in a state of superposition, but some other properties can never be measured precisely simultaneously.

[1] True, already in the Newtonian realm there were known unsolvable problems like the three bodies problem, yet it was held that even these problems were, at least in theory, amenable for numerical calculation provided that measurements can determine the state of the system precisely enough [Eul67, Lag72]. However, that idea was later subverted by Chaos theory [Pui13, page 397], when it was realized that no measurement can be accurate enough, since nearby orbits separate exponentially fast. However, these were operational limits on determinism, ontological determinism was generally accepted.

The quantum measurement, as mediating between the quantum and classic realms, has a central role in the effort to reconcile the two different views of reality. In what follows I will briefly survey the main problems of quantum mechanics and their relation to the measurement problem.

1.1.1 The Discreteness of Energy

One of the first quantum phenomena explored was the discrete spectra of excited atoms. Already at the 17^{th} century the spectra of various materials were investigated and were found to constitute a unique fingerprint for their constituent elements. However, it was only after **Ernest Rutherford**'s discovery of the atom's nucleus [Rut11], enabling physicists to give some explanation to the spectrum, that the discretness became a major obstacle.

It was assumed that the electrons encircle the nucleus in a planetary-like orbits. Since each such orbit has a specific potential energy $E = -\frac{e^2}{r}$ for some radii, transition from high energy orbit to a lower one will result in emission of the excess energy in the form of a photon. However, for the electron being electrically charged, Maxwell's theory of electromagnetism predicted that, due to its continuous acceleration, would emit energy, thereby lowering its orbits until it falls into the nucleus in about 10^{-13} seconds.

In another course of events, by the end of the 19^{th} century, new phenomena were discovered, unexplained by the known physics. One of the most difficult to explain was the **Black Body Radiation**: A Black Body is a perfect body that does not reflect any radiation (hence is black...), it only emits "thermal radiation" which depends *solely* on its temperature. All the attempts to develop a formula to the spectrum of that radiation failed. Furthermore, the calculations predicted that an infinite amount of energy would be emitted by infinitely high frequency photons - a problem known as the **Ultraviolet Catastrophe**.

It was **Max Planck** [Pla01], who managed to give a mathematical description for the Black Body spectrum. In order to do that he conjectured that the radiation originates from a collection of harmonic oscillators having random frequencies that are in equilibrium similar to thermal equilibrium. As in the case at thermodynamics, this assumption was a manifestation of our partial knowledge of the real state of affairs in the microscopic level. However, in order to prevent the UV catastrophe Plank added a discreetness, or quantization, hypothesis: the energy of these resonators must be a whole multiple of some fundamental quantity, now known as **Plank's Constant**, $h \simeq 6.6 \times 10^{-34} Joule \cdot sec$.

By inserting the quantization constraint, Plank managed to prevent the UV catastrophe and to give an exact mathematical description of the Black Body radiation spectrum, but he had no explanation for that discreetness of energy levels.

Trying to explain the discrete spectra of the elements, **Niels Bohr** published in 1913 a model for the atomic structure [Boh13] as what was later known to be the "**Old Quantum Theory**". In this model the electrons were allowed to orbit the nucleus only in certain stable paths called "stationary quantum states". Bohr formulated the properties of the electrons' orbits as a function of the angular momentum, but he had to insert, *ad hoc*, a quantization condition: the only allowed paths are those that make the electron's angular momentum equal to an integer product of $h/2\pi$ (a quantity that was later marked $\hbar$):

$$l = mvr = n\hbar. \quad n = 1, 2, 3, \ldots \tag{1.1}$$

Bohr managed to calculate the discrete lines of the Hydrogen spectrum, assuming transitions of an electron from one allowed orbit to another:

$$\Delta E = (m - n)\hbar = h\nu. \quad m, n = 1, 2, 3, \ldots \tag{1.2}$$

However, Bohr proposed no explanation for this quantization requirement, except for the fact that it accorded with the observed emission spectrum of Hydrogen.

In a parallel course of events, **Albert Einstein** published the article that won him the Noble Prize [Ein35a]. In this article, Einstein explained the **Photoelectric Effect** – an effect in which light shone on a metal generates an electric potential. It was not clear why did the electric potential depend on the *frequency* of the light shone, and not on its *intensity*. Einstein managed to explain that fact by suggesting that the light is composed of undividable quantities, or “quanta” of energy, and not of continuous waves, as was the common knowledge at these days.

1.1.2 Wave–Particle Duality

The strange nature of quantum phenomena begun to become apparent after Einstein published his paper explaining the photoelectric effect. The debate on the nature of light goes a long way back till the days Newton and Huygens, but after **Thomas Young** observed interference [You01] – salient wave-like phenomena – and **James Maxwell** formulated the **Maxwell Laws** that govern the electromagnetic field, it was accepted that electromagnetism is based on waves. Even Einstein, in his paper, notes that he believed so, but in order to explain the photoelectric effect, one has to assume that the energy of light is concentrated in discrete quanta, the **Photon**s:

> The wave theory of light, which operates with continuous spatial functions, has worked well in the representation of purely optical phenomena and will probably never be replaced by another theory...
>
> It seems to me that the observations associated with blackbody radiation, fluorescence, the production of cathode rays by ultraviolet light, and other related phenomena connected with the emission or transformation of light are more readily understood if one assumes that the energy of light is discontinuously distributed in space. In accordance with the assumption to be considered here, the energy of a light ray spreading out from a point source is not continuously distributed over an increasing space but consists of a finite number of energy quanta which are localized at points in space, which move without dividing, and which can only be produced and absorbed as complete units.
>
> *[Ein35a]*

This idea constituted the germ that would grow into the wave mechanics formulation of quantum physics. However that two-faced behavior displayed by light will soon be found to be a hallmark of many starnge quantum mechanical phenomena.

1.1.3 The Probabilistic Nature of Quanta

As Quantum Mechanics evolved, the probabilistic nature was found to characterize all the phenomena it describes. As in the case of Planck's harmonic oscillators, the probabilistic nature was believed to be the result of our incomplete knowledge of the exact internal state of the system – that there are additional "hidden variables" involved, and, should we only knew them, we would be able to predict the exact outcome of the quantum experiments. However, as the theory matured, it became clear that the probabilistic nature is inherent to the quantum world. In 1932 **John von Neumann** proved that there *could not be* such additional hidden variables [vN55, pages 305-324][2]. As a result, when quantum mechanics was finalized, inherent probability was one of its prominent features.

In 1925, when **Werner Heisenberg** gave quantum mechanics its first mathematical form [Hei25], he assumed that every quantum system could exist in one out of a certain number of possible states, with a certain probability attached to each one. That description, the **Matrix Mechanics**, regarded the probabilities as inherent, fundamental, ingredient of the quantum systems, and tried not to give any interpretation nor explanation for that fact.

Later on, Schrödinger developed the wave mechanics and Born gave it a probabilistic interpretation (more details in the next section). The probabilistic nature had to be inserted in an *ad hoc* manner to the measurement process. When von Neumann formulated the Hilbert space formalism, he had to postulate two fundamental processes: One for the time evolution of (isolated) quantum systems, which is strictly deterministic. And a different process that governs the interaction of the quantum system with

[2]Actually, that particular proof was later found to lie on wrong premises. The error has been corrected by Bell [Bel66] but the implications of hidden variables theories are nonetheless acute, see for example [BDH95].

a measurement apparatus. For a system in a state of superposition in regard to the measured property, this process is random, selecting one of the possible states of the system (see also section 1.1.10).

1.1.4 The Wave-Like Nature

In 1923, as part of his Ph.D. Thesis, Prince **Louis de Broglie** [dB25] tried to explain the quantization requirement by considering the possibility that matter particles are basically waves. His reasoning was simple: Since Einstein has shown that light, believed to be undulant phenomena, was also found to present corpuscular behavior, perhaps other corpuscular phenomena, such as matter particles, are linked to undulant behavior. Being waves might also explain the discreteness of energy, since wave equations are notorious for having discrete solutions. In particular, the description of the electron as a wave encircling the nucleus should constrain it to solutions that encircle the nucleus in an integral number of wavelengths, resulting in a discrete energy spectrum.

This theoretical insight leads to an empirical prediction: In a double-slit experiment, electrons, just like photons, should give rise to interference. And indeed an experimental proof followed with Davisson and Germer [DG27] showing diffraction of electrons (a double-slit interference experiment was conducted only in the '60s by Jönsson [Jön61]).

Following de Broglie's suggestion, **Erwin Schrödinger** developed the wave equation associated with his name [Sch26b]. Derived from the Hamilton equation of motion, known from the Newtonian mechanics, the equation described the energy constraints on the system. The solutions of Schrödinger's equation are "wave functions", Ψ, describing oscillations in a complex three-dimensional configuration space. The equation can be solved as an eigenvalue problem, resulting in discrete solutions, corresponding to every time independent state of the system[3]. The counterparts of the physical properties of the classical states are obtained by linear operators acting on Ψ. Schrödinger solved the equation for an electron encircling a hydrogen nucleus. The view expressed in that article, was that the solution is a weight function, in

[3]There are also wave packets which are not eigenstates (*e.g.* in scattering theory).

configuration space, of the electric charge distribution.

However, the accepted interpretation of the wave function was given by **Max Born** soon afterwards [Bor26]. According to this interpretation, the square of the value of the function for any given configuration of the system provides the probability for finding the system at that configuration. It took Schrödinger [Sch26a] and Eckart [Eck26] a short time to prove (independently) that these probabilities are mathematically equivalent to Heisenberg's, in spite of the intrinsic ontological difference between the two views.

One must remember, though, that the naïve interpretation of Schrödinger's Equation of a single particle as a matter wave undulating in the three dimensional position space is misleading. First of all, Ψ is a function that attaches a *complex* number to each point, yet the meaning of complex numbers in a physical theory is not clear. Second, the wave function lives in the **Configuration Space**, that is, the space describing all the possible configurations of all the particles constituting the subject quantum system. Configuration space describing the position of a system's constituents, for example, requires 3 spatial coordinates for *each* particle. When two particles are considered, instead of two functions of three position variables, Ψ becomes a function in *six* spatial variables, providing the (square root of the) probability to find particle 1 at position x_1, y_1, z_1 and particle 2 at position x_2, y_2, z_2.

Despite these conceptual difficulties, Schrödinger's solution was quickly endorsed and numerous experiments [Jön61, CM91, KETP91, ANVA+99, NBAZ01, to name a few] confirmed the fact that what we used to call "matter particles" display, in an appropriately isolated and monitored environment, complex patterns of interference, diffraction and polarization, thereby behaving as waves, in perfect accord with Schrödinger's Equation.

However, one must remember that quantum systems are always measured by detectors that find them at a certain position, never dispersed over a wide region of space, as one would have expected from a wave.[4] Bell even stated that every measurement is actually a position measurement: "...in physics the

[4]Indeed, every measurement, either position or any other physical property, has some spread, but this is due to the

only observations we must consider are position observations, if only the positions of instrument pointers." [Bel87, page 166]. In any case, physical properties, such as mass, momentum, charge, etc., are always found in definite values and never scattered over a broad range, as suggested by the wave equations.[5] Therefore, quantum particles have "split personality": they advance and interact as waves does, but are measured as corpuscles.

It should be noted that a certain possibility to measure the wave function of a single system, without tangibly disturbing it, was described by **Yakir Aharonov** and **Lev Vaidman** [AV93]. The proposed procedure can't construct the phase of the wave, and it calls for a "Protective Measurement", during which the wave function is prevented from substantially changing by weak interactions. In order not to inflict a significant disturbance on the system, a **Weak Measurement** [AV89] should be employed. In a weak measurement, a small bounded disturbance to the quantum system is applied during a long period of time. It was shown that such a measurement does not change the wave function, and the resultant pointer state is proportional to the *expectation* value of the observed property, and not to one of its eigenvalues. The difficulties associated with this hypothetical wave function measurement only highlight the actual impossibility to really know the wave function of a single particle, while the problem of many particle system still remains.

1.1.5 Uncertainty Relations

When **Werner Heisenberg** studied the solutions of Schrödinger's equation, he found a strong relation between a particle's position and momentum. These relations stem from the commutation relation of the operators for measurement of position and momentum. Similar uncertainty relations exist between any properties whose measurement operators are non-commutating [Hei27].

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limits or finiteness of the measurement apparatus. However this spread is usually significantly smaller than the spread of the wave packet.

[5] In any given experimental situation, when repeating some measurement over and over again, different values might be registered. However, in any single experiment there is a single valued result.

At first, Heisenberg tried to explain these relations as technical problem, with the celebrated example of "**Heisenberg's Microscope"**: If one wishes to measure a certain particle's position and momentum, one needs to use a photon that will scatter off the particle. After measuring the scattered photon, the particle's position and momentum could be deduced. However the need to focus the photon to ascertain the particle's position with great accuracy introduces an uncertainty in the momentum measurement, and *vice versa*. Both the position and momentum uncertainties relate to the aperture of the focusing lens, though in a reciprocal way. As a result, trying to decrease one increase the other.

It is worth mentioning that the above explanation for the uncertainty relations is classical, that is, it *presumes the existence of position and momentum at any given moment of time*, the difficulty of measuring these properties being merely technical. However, the quantum mechanical formalism *cannot* describe both exact position and exact momentum at the same time. That means that as much as quantum mechanics represents reality, this impossibility is ontological and not a result of some kind of lack of knowledge (epistemology) or poor technological capability, as Heisenberg's Microscope implies.

Accepting the fact that in quantum theory it is not possible to describe both position and momentum of particles actually makes it impossible to call these entities "particles". In classical physics, the description of particles is through the relation between position and momentum (*e.g.* phase space, Hamilton and Lagrange equations). Accepting the quantum uncertainty relations calls for totally different entities at the microscopic scale. Though still called "particles," they don't have the properties one would expect classically, but they are still particles in the sense that they retain their identity in the macroscopic world.

Bohr was probably the first to fully understand that point, insisting that the uncertainty relations articulate a profound truth. He rooted the impossibility on the fact that any experimental setup can measure precisely only one of these properties, while increasing the uncertainty in the other. Bohr elaborated this "in principle" impossibility of simultaneously measuring couples of non-commuting variables into broader ideas such as "Complementarity" and "**Duality**":

Thus, a sentence like "we cannot know both the momentum and position of an atomic object" raises at once questions as to the physical reality of two such attributes of the object, which can be answered only by referring to the unambiguous use of space-time concepts, on the one hand, and dynamical conservation laws, on the other hand. While the combination of these concepts into a single picture of a casual chain of events is the essence of the classical mechanics, room for regularities beyond the grasp of such a description is just afforded by the circumstance that the study of the complementarity phenomena demands mutually exclusive experimental arrangements.

[Boh58, pages 40-41]

1.1.6 Complementarity

The concept of Complementarity was introduced by Bohr at the International Congress of Physics at Como [Boh28] and was vague from the very beginning. Bohr viewed the true behavior of microscopic particles as distinctly different from the behavior of the macroscopic measurement systems. Consequently, when one uses different measurement apparata to examine different aspects of the quantum behavior, the joint interaction of the quantum system *and* the measurement apparatus results in phenomena that apparently contradict each other. However the contradiction is only in the phenomena resulting from the utilization of different measurement systems and not in the quantum system itself, and yet we cannot get a complete understanding of the quantum behavior without resorting to measurements of both types of phenomena.

Bohr stated:

"Consequently, evidence obtained under different experimental conditions cannot be comprehended within a single picture, but must be regarded as complementary in the sense that only the totality of the phenomena exhausts the possible information about the objects. ... the study of the complementary phenomena demands mutually exclusive experimental arrangements"

[Boh58, page 40]

Bohr also applied his idea of complementarity to the wave-particle duality. He argued that whether a quantum particle behaves as a wave or as a corpuscle depends on the whole of the experimental setup. By so saying, he ignored the fact that even when the experimental settings is planned to measure undulant behavior, every particle will eventually be measured in a specific point – a prominent corpuscle method. The interference pattern will appear only in intensity calculations summing a large numbers of experiments: even though each particle is measured in a certain point, only the sum of many hits reveals the interference pattern.

Another fact that Bohr ignored is that complementary properties (even being a wave or a particle) are not parts of an either-or dichotomy, but two ends of a continuous spectrum, and it is possible to gain partial knowledge of one, at the expense of the other. A good example is the following simple experiment: in a Young (double-slit) experiment, put a horizontal polarizer in front of one slit and a vertical one in front of the other. The which-path information gained in that way will force the photons to show their "corpuscular" face, thereby destroying the interference pattern on the screen. However, rotating one of the polarizers towards the angle of the other will make the distinction even less clear, hence "wash out" the corpuscular identity of the photons, gradually restoring the interference pattern (see a similar setup in [WCPM02]).

1.1.7 Superposition

One of the distinct "bi-products" of the wave-like nature of quantum particles is the existence of Superposition. Since Schrödinger's equation is linear, if ψ_1 and ψ_2 are valid solutions for some experimental setup, so is their sum $\psi_1 + \psi_2$. Since the quantum states harbor a complex phase, measuring these sums result in a constructive and destructive interference, leading to all the odd phenomena identifying quantum processes.

In essence, superposition is *the* phenomenon separating quantum systems from macroscopic behavior. On one hand, the interference caused by superposition is the first visible result obtained in experiments. On the other hand, though, a measurement of a quantum system in a state of superposition will never give a superposition of the possible results, since that requires the measurement apparatus (which is macroscopic and hence classical) to be in itself in a superposition state, displaying two (or more) distinct results at the same time.

From the philosophical point of view, superposition presents great difficulties: What does it mean that a quantum particle can be measured in ψ_1, ψ_2, or ψ_n? Is it really in the n states at the same time, or maybe it actually is in one of the states, but we, the observers, still don't know that, and the wave function merely reflects our lack of knowledge? And even if we suspend our judgment in regard to quantum particles, the theory can't easily "get rid" of the superposition when constructing the macroscopic world out of quantum particles. See the discussion in Section 1.1.10.

1.1.8 Non-Locality

The most simple non-local behavior displayed by quantum systems was noticed already at 1927 by Einstein [Boh58, page 42] – quantum particles may spread as waves across a wide region, but they are measured as point-like particles. At the moment of detection, the probability for them to exist in other regions of the wave-front is instantaneously, and non-locally, nullified. Since (according to the quantum postulates) the reduction is immediate, and permeates the whole universe, it has an apparent non local nature. It also breaks causality as defined by the Special Theory of Relativity). In general, the reduction of the quantum state (section 1.1.10) exerts non-local correlations on different parts of a quantum system, which might be arbitrarily far apart.

The more modern notion of non-locally involves **entangled** systems. According to the quantum theory, when quantum systems interact they become dependent, or *entangled* (entanglement can also occur when the initial conditions tie several systems together, *e.g.* the sum of spins of two particles equals zero). In such a case, the results of measurements performed on one system depend on the state of another. Formally, the two systems cease to exist in a product, or separable state, where each one can be described by a state in a separate Hilbert space. The two systems can now be described only by a state residing in the tensor product of the two Hilbert spaces. Entanglement causes a non-local **Contextuality**, that is, a measurement on one system depends on the state of affairs on the other, which might lie far apart. As will be elaborated later (section 1.2), these non-local results are inherent to the quantum entanglement and cannot be ascribed to any predetermined local hidden variables.

At first glance it might look as if a "spooky action at a distance" is taking place, in contradiction to the Special Theory of Relativity. However, farther examination [Ebe78] proved that there is no way to transmit information between distant parties in that way. Quantum mechanics shows that even for entangled systems, measurement results on each party are random, in perfect accordance with quantum theory. No way exists allowing one party to change the probabilities of measurement results at the other party. Only when the measurement results are brought together and compared, certain patterns of correlation are revealed. However, these patterns cannot be controlled, nor revealed or examined until the distant parties communicate in sub-luminal way.

1.1.9 The Problem of Preferred Basis

Every quantum state, $|\Psi\rangle$, can be described as a linear combination of some set of orthogonal vectors. The set of vectors is called a basis. Denoting the basis as $\{|\alpha_i\rangle\}$, a state $|\Psi\rangle$ will be written as:

$$|\Psi\rangle = \sum_i c_i |\alpha_i\rangle, \tag{1.3}$$

where $\{c_i\}$ are the coefficients which uniquely describe $|\Psi\rangle$. For a continuous basis, the sum is replaced by an integral.

For a non-degenerate observable operator A, the set of eigenstates solving the eigenvalue equation

$$A|\alpha_a\rangle = a|\alpha_a\rangle \tag{1.4}$$

comprises such a basis. For example, the eigenstates of the position operator, x, form a set of generalized eigenstates in the continuum, hence any state $|\Psi\rangle$ can be described as:

$$|\Psi\rangle = \int_x a(x)|x\rangle \; dx. \tag{1.5}$$

However, the eigenstates of the momentum operator, p, present a basis as well. That means that the same state can be equally written in the momentum basis:

$$|\Psi\rangle = \int_p b(p)|p\rangle \; dp. \tag{1.6}$$

When a measurement of a certain operator is performed, one of its eigenstates is selected to be the measured one, thereby making the operator value-definite. Usually, if not always, the measurement boils down to a value-definite position of some pointer and if not a pointer, then position of electrons in a computer's memory or atoms in the Rhodopsin molecule in our retina (*cf.* citation from [Bel87] on page 11). In a collapse or hidden-variables interpretations of quantum mechanics, this fact is obvious. However, in non-collapse interpretations, such as **Hugh Everett**'s **Many-Worlds** [Eve57], this is a very strange result – why should we observe value definite pointers in the position basis, and not in the momentum basis, for example? In other words, why is the position basis a preferred one?

One idea which was conceived in order to circumvent this problem is the theory of **Decoherence**, advocated by **Wojciech Zurek** [Zur81, Zur82]. According to this theory when the environment interacts with a quantum system, a natural decay occurs which quickly brings the system to a state *similar* to a diagonalized density matrix of the position operator – a state of definite position. Zurek showed such an effect when the environment was represented by a collection of harmonic oscillators in a thermal equilibrium. That means, that *if* the environment is really similar to Zurek's assumption, any quantum system that will interact with it will transform into a state that, for any practical purpose, has a definite position. Note that this interpretation requires that the environment interact with the quantum system in a very specific manner, a condition that is not always met. Apart from that, many reservations were raised against the theory, some of them were summarized by Adler [Adl03].

1.1.10 The Measurement Problem

As mentioned earlier, quantum particles seem to present a two faced behavior: their dynamics fits a wave-like behavior, while they are always measured as point particles.[6] And there are a few additional mysteries wreathing the measurement process:

- Quantum particles, as a result of their wave-like nature, can reside in a state of superposition. But upon measurement, superposition is never observed.
- At the moment of measurement, the state of a quantum system is abruptly changed in a process called "**Collapse**", or "**Reduction**" of the wave function. This change does not conform to Schrödinger's equation, is not deterministic, and not reversible. Let a quantum system be measured for a certain observable property, A. Before measurement, it was in a state of superposition of several possible values of A: $a_1, a_2, ...a_n$ (possibly an infinite set of continuous values). After the measurement process, the system will be in one of the eigenstates of A, a_i. The measurement process is indeterministic since there is no way to predict which of the possible states the system will "collapse" into. The measurement is irreversible since there is no way to figure out the initial superposition given the measurement result.
- Sometimes the measurement seems to fix history *backwards*. The most prominent example is **John Wheeler**'s Delayed Choice experiment (to be discussed later in Section 6.2) in which a choice of different measurements *at the end* of the experiment imply different descriptions for the state of affairs *at the midst* of the experiment. Furthermore, in the **Two-Vector** [ABL64, RA95], and the **Transactional** [Cra86] interpretations, the future interacts back with the past in order to establish measurements' results. I will later demonstrate thought experiments in which a measurements seem to exert noticeable effects backward in time (Chapters 5, 6). If this view is valid, it will have a profound impact on our understanding of time's nature.

[6] It is possible to measure the radii of protons, neutrons, and mesons. However, these are composite particles, composed of a couples or triplets of quarks. The quarks and leptons themselves are considered elementary, point like, particles.

So the standard formalism of Quantum Mechanics calls for two distinct dynamics: one describes how quantum systems evolve and interact, obeying Schrödinger's equation, travelling about as waves, constructing superpositions, and displaying patterns of interference. But then there is the measurement process, where the wave function swiftly "collapses" non-linearly, non-locally, and without preservation of unitarity,[7] to one of the eigenvalues of the measurement operator.

It looks as if the interaction with the measurement apparatus is fundamentally different from the interaction between quantum systems, even though the basic assumption of physics is that macroscopic entities are comprised of the microscopic entities – quantum systems. What is it then, that fundamental difference, between the particles that are tested in quantum mechanical experiment and their siblings, comprising the measurement apparatus?

Moreover, it seems that the measurement problem lies at the heart of all the problems mentioned earlier:

- The probabilistic nature appears only during the measurement process. The existence of interference suggests that before the measurement, when in superposition, the particles were in all possible states. The reduction of the state into one of these possibilities happens only when the particles interact with a macroscopic measurement apparatus.

- The wave-like nature of quantum particles disturbs us mainly because of the wave-particle duality, that is, the fact that quantum particles seem to be measured as a point-like corpuscles. Without the reduction of the quantum state, we could have regarded single particle quantum systems as waves obeying Schrödinger's equation[8]...

- The problem with superposition is that, because of the reduction at the measurement process, they never appear in the measurement results. We can infer the existence of superposition only from secondary observations, *e.g.* interference patterns.

[7] There are models of collapse that do preserve unitarity, though, *cf.* Ghirardi, Pearle and Rimini [GPR90].

[8] Note that in such a world, our interaction with quantum particles will not be blind to their phase...

- As was mentioned above, the preferred basis problem is also related to the measurement process.
- The non-locality of quantum phenomena appears only during the swift reduction of the quantum state. That instantaneous nature is a source for numerous conceptual problems in relativistic quantum mechanics [AA80, AA81].

The measurement problem demarcates the border between macroscopic and microscopic, classical and quantum, particle and wave. From all the above, it's apparent that the measurement problem epitomizes the mystery of Quantum Mechanics.[9] It breaks the beauty of the theory's formalism and is the main reason for quantum particles to behave so different from the macroscopic entities we know in our everyday experience.

For these reasons I will concentrate this thesis on the measurement problem and its investigation. I will describe various gedanken experiments designed to explore the problem from different angles. I hope that the investigation will endow us with some new insights concerning issues as diverse as wave-particle duality, the non-local behavior of the quantum state, information transfer and its limits, the extent of the notion of realism in the quantum state, and the definition and role of causality in quantum mechanics.

1.2 The Debate over Realism in Quantum Mechanics

The first philosophical question regarding quantum mechanics concerns its ontology: what is the nature of the fundamental entities described by Quantum Mechanics? The common knowledge is that ontology resides in the realm of philosophy, since it deals with pre-scientific notions, hence it can only affect the scientific theories and never be affected by it. However, quantum mechanics provided us with one of the most beautiful cases where a scientific theory managed to directly affect a philosophical dispute. The

[9] More precisely, it is the existence of incompatible variables (like position and momentum) that epitomizes the mystery of quantum mechanics, but this says that such variables cannot be measured simultaneously, and it is this element that constitutes the fundamental basis of the measurement problem.

argument between Einstein and Bohr has begun as a philosophical argument, but ended up with an experimental resolution.

As early as on the first days of quantum mechanics, arguments over its fundamental concepts and the possible interpretations of the maturing formalism began. The most famous arguments took place between Einstein and Bohr, during the years 1927-1930 [Boh58, pages 41-58]. Einstein, still fixated on the Newtonian, realist world view, believed that all those problems mentioned earlier, which place limits on our ability to know reality, reflect the limitations of the theory and not of the phenomena involved. He then tried to prove, by means of *Gedanken* (thought) *Experiments* that there are parts of reality that are inaccessible to quantum mechanics, thereby proving the latter incomplete.

Bohr, on the other hand, accepted the quantum formalism literally, with all of its derived consequences. As a result he adopted a "positivist", or "operationalist" approach and denied any possibility to even discuss the world *an sich* (in itself). For him, the only facts a scientist can hold on to are experimental results [Boh28].

Bohr's stance was holistic, even Aristotelian [Bec99, chapter 3.12]. By his own words,

> The crucial point... implies the *impossibility of any sharp separation between the behavior of atomic objects and the interaction with the measuring instrument which serve to define the conditions under which the phenomenon appear...* any attempt to subdividing the phenomena will demand a change in the experimental arrangement introducing new possibilities of interaction between objects and measuring instruments which in principle cannot be controlled.
>
> *[Boh58, page 39, italics in the original]*

According to this view, it was pointless to talk about quantum particles or about their properties *per se*, before a proper measurement was performed. This position emerged mainly out of a clear understanding of the quantum formulation, where the measurement operation "selects" one result out of several possible ones. If the quantum formalism reflects ontology in any way, then one cannot describe the system as having any of the possible states prior to measurement. Bohr chose to interpret that as if before the measurement there is only a *potential* for the possible results – but still no real existence for none of them – and at the measurement process one result becomes real.

EPR Experiment

A major milestone in that argument was the article written by Einstein, Podolsky, and Rosen [EPR35], known by the name of EPR. In that paper they practically laid down the foundation to all the arguments in that research area up to now.

In their paper, EPR presented a realist's postulate, which I will hereforth take to be the definition for the realism of any physical quantity: "If, without in any way disturbing a system, one can predict with certainty the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity". At first glance, that assumption seems like a plausible premise of every scientific discussion. Equipped with that argument, they presented a system of two entangled particles which seemingly allows predicting two non-commuting properties of one of the particles by performing measurements on the other, distant, particle. Since EPR denied action at a distance, they concluded that there exists an element of reality corresponding to the two non-commuting properties – a fact that cannot be accounted for in the quantum theory.

Bohr quickly responded [Boh35] that there are fundamental flaws in EPR's realism assumptions: from his point of view, there was no meaning for discussing properties of a quantum system prior to its measurement. Moreover, measurement results apply for that system with this specific experimental setting. Changing the measurement apparatus and conducting another measurement might void our prior knowledge about the system's state.

Thus, two experiments where two non-commuting properties are measured cannot be compared with each other – even though they were carried out in succession. Each experiment measures a complementary property of the system, each of the two results reveal only part of reality.

Eventually, it was believed that the question of the realism of quantum mechanics is undecideble within the scientific realm. It is bound to the free, pre-scientific, choice of notions like "element of reality" that is made by each individual, and cannot be proved nor disproved by experiments.

Bell's Inequality

All that changed in the 1960's, when **John Bell** realized that it was possible to resolve the debate experimentally, and to transfer the question from metaphysics to the scientific arena. Bell formulated an **inequality** [Bel64] that draws limits on measurements' results of every possible **local-realistic** theory (that is, a theory with real physical entities, and local interactions). Namely, this inequality must be met by any theory that foresees particles that carry their own properties with them, and cannot move nor communicate superluminally. It appears that quantum mechanics predicts *violations* of this inequality should it be tested experimentally!

Only in the 1980's, a series of experiments [AGR81b, AGR81a, ADR82, KMWZ95, TBZG98] vindicated quantum mechanics: Local Realism was refuted by experiment. As long as no loopholes are found in Bell's proof, *every* theory that reproduces the (experimentally confirmed) quantum probabilities, cannot be both realistic and local.[10]

The main result of Bell's proof was that entangled systems display correlations that cannot be explained by assuming some previous agreement on the results of all possible experiments (in the form of local, hidden, variables, for example) between the distant entangled parts of the system. That means that the results of measurements on one part of an entangled system depend on its context, that is, measurements exerted on some distant parts of the system. This property is called **Contextuality**. Therefore, in an EPR like setup for example, the result of a measurement on one particle depends on the measurement carried out on its sibling. This conclusion will be of importance later on, in Section 6.7.

[10] Loopholes were proposed in the *experiments* that checked Bell's inequality violation [Fra85], but correction were proposed to overcome them [KESC94] and further experiments carried out [ea98, RKM+01].

Farther Limits on Realism

Making the realistic view even more difficult, **Simon Kochen** and **Ernst Specker** [KS67] found yet another limit emerging from the quantum theory. In their paper, they refute (for systems in a three dimensional Hilbert space or higher) the **Value Definiteness** premise, according to which it is possible to give definite values to all observables defined for a quantum mechanical system, at all times. Their proof was based on geometrical properties of the Hilbert space. They showed that if physical properties are represented as operators on a Hilbert space, then these properties cannot all be said to simultaneously have values. They managed to show that for a specific set of 117 observables, trying to assign definite values simultaneously causes a contradiction. Further formulations of this idea reduced the maximal number of concurrent observables to 33 [Per91], and even 20 (in 4 dimensional Hilbert space) [Ker94].

The next step was carried out by **Daniel Greenberger**, **Michael Horn**, and **Anton Zeilinger** [GHZ89], who disposed of the statistical nature of Bell's proof. They proposed an experiment with three entangled spin $^1/_2$ particles. In their setup, measuring two of the particles gives a certain prediction for the results of a measurement of the third. It was shown that a certain set of 4 such measurements yields a counterfactual statement that cannot be met by any local realistic theory. Such an experiment was performed [BPD$^+$99] and its results agreed, again, with the quantum mechanical predictions.

Later on, **David Mermin** [Mer90] reached a better limit on hidden variables by showing that for two spin $^1/_2$ particles, no hidden variables can be simultaneously assigned to the following nonet of observables:

$$\begin{array}{ccc} \sigma_x^1 & \sigma_x^2 & \sigma_x^1\sigma_x^2 \\ \sigma_y^1 & \sigma_y^2 & \sigma_y^1\sigma_y^2 \\ \sigma_x^1\sigma_y^2 & \sigma_y^1\sigma_x^2 & \sigma_z^1\sigma_z^2 \end{array}$$

GHZ provided a refutation to local realism, resorting neither to statistical analysis nor to inequalities, however, they required three particles. Mermin used two particles but nine observables. **Lucien Hardy** [Har92b, Har93a] managed to disprove local realism by using only two particles. His proof will be discussed in Section 2.1.1.

1.3 A Note on Thought Experiments

As mentioned before, **Gedanken** (thought) **Experiments** had an important role in the development of science and philosophy. The first ideas can he found already in the pre-Socratic era, such as the Zeno paradoxes [Hug99]. Then they were found through the dawn of science with examples like Galileo's refutation of the Aristotelian theory of gravity [Gal83], Newton's "Bucket Argument" proving the existence of absolute space [New87], and Maxwell's Demon [Max71]. The revolutions of the 20^{th} century brought many more, like Einstein's attempts to disprove quantum mechanics in the early Solvay conferences [Boh58, pages 41-58], the EPR argument [EPR35] that showed the limits of quantum mechanics, and Einstein's "Hole Argument" that helped him to formulate the General Theory of Relativity [EN87].

In the following chapters I will follow this distinguished tradition and introduce a few novel gedanken experiments in order to investigate the fundamental concepts of quantum mechanics and to highlight their strange consequences. As mentioned before, I will concentrate on the measurement process and study it from various angles. I will make use of different kinds of measurement processes: Partial Measurement, Interaction Free Measurement, and Quantum Erasure.

As in every gedanken experiment, which is supposed to highlight subtle points in a scientific theory, I will introduce experiments aiming at the process of measurement. Such experiments need not be actually carried out, since they do not intend to test a specific facet of the physical theory. Instead they intend to emphasize the consequences of accepting the quantum mechanical formalism "as is".

In any case, a good thought experiment is worth carrying out, if only for its demonstrative and educational value. I hope the experiments that follow will attract the attention of an experimentalist who will take the labor to carry them out.

1.3.1 A couple of technical remarks

In order to simplify the analysis, I have used an apparatus that employs photons and atoms with the simplest interaction possible: If the atom's position turns out to be on the photon's path, there is a 100% probability for absorption and detection of the photon. And, of course, if the atom turns out not to be in the photon's path, no interaction will take place. I did not use scattering matrices to calculate scattering phase shifts since, under these idealized conditions, either there is no interaction, or a complete capture of the photon ensue. The experiments are intended to investigate the strange implications of superposition and measurement, hence, in this framework, it would be impractical to add further complication in the form of photon scattering calculation.

Of course, a real experiment will have to take into account a vast group of technical considerations, such as interaction cross section, scattering, detector efficiency etc. However, the theoretical literature on quantum measurement [Har92b, Har92a, EV93, RAPV95, to name a few] has made an extensive use of such highly simplified setups, reasoning that once the basic interactions are well understood, the experimenter who eventually performs these experiments would take care of the other realistic factors involved. This, indeed, was the case with Zeilinger *et. al.* 's celebrated experiments [BPM+97, BPD+99, ANVA+99, NBAZ01], that were based on highly idealized thought experiments.

As another mean for simplification of the analysis, I have used time-independent formulation for the wave function. The overall assumption is that the governing Hamiltonian is degenerate in respect to the analyzed properties (usually spin direction or photon polarization), hence no phase difference will accumulate between different eigenstates of the property. Consequently, it is possible to omit a global multiplication by the term e^{-iEt}.

These two assumptions are commonplace in thought experiments. A good example is Aharonov and Vaidman's three boxes paradox. The original idea was presented in [AV91] and [Vai96]. The two simplifying assumptions presented above were applied in these two papers. Nevertheless, in 2003 an experiment was conducted [RLS04] demonstrating the paradox using quantum optics techniques. Note that even this paper did not go into specifying scattering phases or the overall e^{-iEt} factor.

Chapter 2

Interaction Free Measurement and its Recent Variants

"Thus, the task is, not so much to see what no one has yet seen, but to think what nobody has yet thought, about that which everybody sees".

Erwin Schrödinger

One of the tools I will employ in order to explore the measurement process is the **Interaction Free Measurement** (IFM). The first clues for the fact that even "negative-measurement" modifies the state of the observed system rose already at the 1950s [Ren53], but only in the 1990s **Avshalom Elitzur** and **Lev Vaidman** came up with the exact idea [EV93].

In their article, Elitzur and Vaidman showed that when a particle is measured, even when the measurement ends without an interaction (the detector doesn't "click"), the wave function changes, and this change is measurable. Moreover: such a change can teach us about the measurement process. In their article they investigated a **Mach-Zehnder Interferometer** (MZI) as depicted in Figure 2.1. A single-

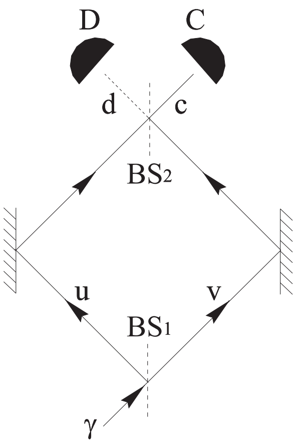


Figure 2.1: Mach-Zehnder Interferometer.

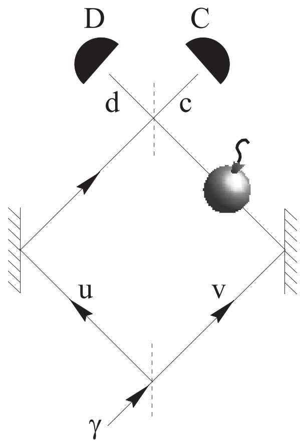


Figure 2.2: Interaction Free Measurement.

photon source emits a γ photon towards a half-reflecting Beam Splitter BS_1. The two beams are then reflected back by two mirrors so as to interfere on a second BS, BS_2. By appropriately fixing the optical paths one can achieve a complete destructive interference on path d, causing *all* the photons to emerge out of the MZI on path c.

Technically, the photon was split by BS_1:

$$|\Psi\rangle = |\gamma\rangle \xrightarrow{BS_1} \frac{1}{\sqrt{2}}(i|u\rangle + |v\rangle). \tag{2.1}$$

The multiplication of $|u\rangle$ by i is due to phase rotation resulting from the reflection.[1] BS_2 then brings about the following transformation:

$$|v\rangle \xrightarrow{BS_2} \frac{1}{\sqrt{2}}(|d\rangle + i|c\rangle) \tag{2.2}$$

$$|u\rangle \xrightarrow{BS_2} \frac{1}{\sqrt{2}}(|c\rangle + i|d\rangle), \tag{2.3}$$

[1] The π phase shift upon reflection results from Fresnel equations for reflection and refraction [FLS65, Volume II, Chapter 33]. When calculating the reflection coefficient R (which is the ratio of the reflected electric field to the incident one – $\frac{E_{reflected}}{E_{incident}}$), one sees that if the refractive index of the first material is smaller than the one for the second ($n_1 < n_2$), then the angle of incidence will be higher than the angle of transmission ($\theta_i > \theta_t$), that, in turn, will cause R to take a negative value – which means a phase shift of π between the incident and the reflected beams.

arriving to the result:

$$|\Psi\rangle \xrightarrow{BS_2} |c\rangle. \tag{2.4}$$

This result means that interference at the second beam splitter causes each and every photon that enters the MZI to leave it on path c. That will occur in spite of the fact that without the interference, BS_2 has an equal probability to emit the photon either on path c or on path d.

Another interesting fact is that the interference at the beam splitter will occur although the source emits photons one at a time. The fact that all measurements are registered only at detector C proves that *each individual photon undergoes interference.* Somehow, the photon actually traveled through both paths u and v. That means that blocking one of the paths will result in destroying the interference pattern at the exit of the MZI. That is the very heart of the IFM idea.

IFM, as was demonstrated by Elitzur and Vaidman (EV), is achieved when a macroscopic detector is presented onto path v – a super sensitive bomb in EV's article (see Figure 2.2). The fact that a detector is present on path v can result in one of two outcomes:

1. The detector registers a particle (or the bomb explodes), meaning that the photon went through path v and was detected there. In that case, the wave function becomes:

$$|\Psi\rangle \xrightarrow{detection} \frac{1}{\sqrt{2}}|v\rangle. \tag{2.5}$$

2. The detector doesn't click (or the bomb remains intact). Although that doesn't qualify as a measurement in the ordinary sense, it *does* affect the photon's state, an influence that has measurable results. Since the photon was not found in v, it must have gone through u, hence the state becomes:

$$|\Psi\rangle \xrightarrow{no-detection} \frac{i}{\sqrt{2}}|u\rangle, \tag{2.6}$$

and the interference pattern, at the output of the MZI, breaks:

$$|\Psi\rangle \xrightarrow{BS_2} \frac{1}{2}(i|c\rangle - |d\rangle). \tag{2.7}$$

Here, there are 50-50 chances that detector D will click.

That result means the following: by considering only the measurement results from the D detector at the *output* of the interferometer, one can determine whether a measurement was carried out *inside* it, while *no interaction* whatsoever took place.

In a real experiment one would have to make sure that the photon would not have bypassed the "bomb" detector (maybe by placing an opaque screen through the MZI leaving only a small hole for path u). Another problem that an experimenter will have to deal with is the efficiency of the detectors: less than 100% efficiency will skew the results. However, when dealing with thought experiment, one is free of such worries – in the above analysis we assumed the detector completely blocked path v, and all the detectors to be 100% efficient.

EV article gave rise to a surge of research both theoretical and experimental [Har92b, Gri93, DHS93, Pen94, KWH$^+$95, to name a few]. Some are reviewed below, see [Vai00] for a comprehensive analysis. I see the novelty in this experiment in the fact that in contrast to the usual measurement process, where a classical detector measures a quantum one, here, a quantum particle effectively measures a classical apparatus: it's the photon, in a state of superposition, that went through the MZI, and upon leaving BS_2 carried information regarding the whereabouts of the classical detector.

The important lesson revealed by these experiments, in my view, is the fact that the quantum particles in the measurement process do not behave like corpuscles, nor like macroscopic waves. They behave in a strange and alien way that does not comply with the everyday notions we know from our macroscopic experience.

2.1 When the detector is also quantum

In their paper, EV mentioned the possibility of an IFM in which both objects, the measuring and the measured, are single particles, in which case even more intriguing effects can appear. This proposition was taken up in a seminal paper by Hardy [Har92a] in which he studied the question: what will happen when the measurement device (the "bomb") will be, too, a quantum device in a state of superposition? The results are truly remarkable and are as follows (the setup is slightly modified to fit the structure of later experiments in this thesis, however the fundamental idea is preserved) see Figure 2.3.

Consider an EV device where a single photon traverses an MZI and interacts with an atom in the following way: A spin $^1/_2$ atom is prepared in a spin state $|X^+\rangle$ (that is, $\sigma_x = +1$), and split by a non-uniform magnetic field M into its two Z spin components. The box is then carefully split into two halves, each containing either the $|Z^+\rangle$ or the $|Z^-\rangle$ part, while preserving their superposition state:

$$\Psi = |\gamma\rangle \cdot \frac{1}{\sqrt{2}}(|Z^+\rangle + |Z^-\rangle). \tag{2.8}$$

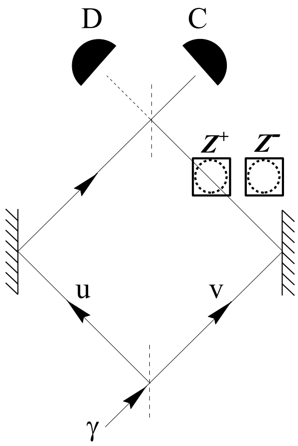


Figure 2.3: A photon in a Mach-Zehnder Interferometer interacting with a superposed atom.

Now let the photon be split by BS_1:

$$\Psi = \frac{1}{2}(i|u\rangle + |v\rangle) \cdot (|Z^+\rangle + |Z^-\rangle). \tag{2.9}$$

The boxes are transparent for the photon but opaque for the atom and keep it insulated from the rest of the environment thereby retaining the superposition intact. The atom's Z^+ box is positioned across the photon's v path in such a way that the photon can pass through the box and interact with the atom inside. Let us also assume that should the photon hit the atom, it will be scattered in a 100% efficiency, removing the term $|v\rangle|Z^+\rangle$ (25% of the cases), leaving:

$$\begin{aligned}\Psi &= \frac{1}{2}(i|u\rangle|Z^+\rangle + i|u\rangle|Z^-\rangle + |v\rangle|Z^-\rangle) \\ &\quad +|scattering\rangle. \end{aligned} \tag{2.10}$$

In what follows, we will post-select out the scattering events. That is, from any set of experiments, 25% will end up in scattering and will be ignored. The rest of the analysis pertains only to the 75% of the experiments that did not end up with scattering.

Let us now reunite the photon by BS_2:

$$|v\rangle \xrightarrow{BS_2} \frac{1}{\sqrt{2}}(|d\rangle + i|c\rangle) \tag{2.11}$$

$$|u\rangle \xrightarrow{BS_2} \frac{1}{\sqrt{2}}(|c\rangle + i|d\rangle), \tag{2.12}$$

so that

$$\Psi = \frac{1}{\sqrt{2}^3} \cdot \big(i|c\rangle(|Z^+\rangle + 2|Z^-\rangle) - |d\rangle|Z^+\rangle\big). \tag{2.13}$$

Once the photon reaches one of the detectors, the atom's state is recombined by a reverse magnetic field $-M$.[2] The state is then:

$$\begin{aligned}\Psi &= \frac{1}{4}|c\rangle \cdot (3|X^+\rangle - |X^-\rangle) \\ &\quad -\frac{1}{4}|d\rangle \cdot (|X^+\rangle + |X^-\rangle).\end{aligned} \tag{2.14}$$

Here, measuring the position of the atom, might end up with the photon hitting detector D, while the atom is found in a final spin state of $|X^-\rangle$ rather than its initial state $|X^+\rangle$. In such a case, *both particles performed IFM on one another*, destroying each other's interference. Nevertheless, the photon has not been scattered, so no interaction between the photon and the atom seems to have taken place.

Hardy's analysis revealed the striking consequence of this result: The atom can be regarded as EV's "bomb" as long as it is in a superposition, whereas a measurement that forces it to assume a definite z spin (to "collapse") amounts to "detonating" it. However, the photon's hitting detector D indicates that it has been disturbed too (since if it wasn't disturbed, the interference would have caused 100% of the photons to reach detector C). And yet, in the absence of scattering, no interaction has occurred between

[2] Note that this is a *gedanken* experiment, hence it is possible to assume such a process, which is theoretically valid but very difficult to achieve experimentally.

the photon and the atom. That seems to indicate that the photon has traversed the u arm of the MZI while "detonating" the atom on the other arm (v), forcing it to assume (as measurement would indeed confirms) a definite Z^+ spin!

The analyses of this result were many and diverse: Hardy himself argued that this case supports the guide-wave interpretation of quantum mechanics [Har92a, Har93b]. His reasoning was that the photon's corpuscle-plus-half-wave took the u arm of the MZI while its other, empty half-wave took the v arm and broke the atom's superposition. However, Clifton [CN92] and Pagonis [Pag92] argued that the result is no less consistent with the "collapse" interpretation. Griffiths [Gri93], employing the "consistent histories" interpretation, argued that the result indicates that the particle might have taken the v arm as well, and Dewdney *et. al.* [DHS93] reached the same conclusion using Bohmian mechanics.

In contrast to these, I think that the most important point in both the original IFM experiment and the latter one is the emphasis that the latter puts on the act of measurement. Until the moment of measurement the system resided in a state of superposition which was clear and understandable (though alien to our everyday experience). However, the act of measurement, which caused the immediate reduction of the quantum state, introduces the oddity into the situation.

I shall continue exploring these ideas on Chapter 5.

2.1.1 Refuting Local-Realism (again)

One important result obtained from the previous setup is the simplest refutation of local realism. One can translate Hardy's argument to the setup presented in Figure 2.3 is as follows: assuming local realism entails some kind of internal (hidden) variable that determines the state of the photon and the atom. Its conjectured value must comply with the measured results of both the photon and the atom, and the locality condition forbids any measurement done on one particle to influence the other. The argument calls for measuring the photon either at the paths u and v or, after interfering, at C and D; and measuring the spin of the atom either in the z or the x direction.

There are several conditions that must be met:

1. We will never find, in the post-selected ensemble, that the photon is at route v and the atom at box $|Z^+\rangle$ (because such a coincidence will cause a scattering, which we post-select out).

2. If the photon reached detector D, something must have interrupted it (a non-interrupted MZI will *always* click at detector C), hence measuring the atom's z spin must reveal it in the intersecting box $|Z^+\rangle$.

3. If the atom is found in a $|X^-\rangle$ spin (in contrast to its initial $|X^+\rangle$ state), something must have interacted with it, hence the photon must be found in the intersecting path v, if measured before the interference BS.

4. However, as seen in Equation 2.14, in $^1/_{16}$ of the experiments, both the photon will hit detector D, and the atom will be found in $|X^-\rangle$ spin.

These four conditions cannot be met together. (4) and (3) entails that in $^1/_{16}$ of the experiments the photon must have been found in route v if measured before interfering. Adding condition (2) requires that the atom must have been in $|Z^+\rangle$ if measured. But (1) forbids the atom to be in the $|Z^+\rangle$ box when the photon is in the v route.

The conclusion is that a strong contextuality holds here, such that the results of measurements on the atom are affected by the kind of measurements performed on the spacelike separated photon (or *vice versa*). Local realism is thus refuted.

2.2 Entangling Distant Atoms

In another experiment [Har92a], Hardy has elegantly integrated the peculiarities of the EPR experiment, single-particle interference and the interaction-free measurement – all in one simple setting.

Let a single photon traverse a MZI. Let two spin $^1/_2$ atoms be prepared as in the previous experiment (Figure 2.4): Each atom is prepared in an $|X^+\rangle$ state, and then split into its σ_z spin components. The two components are then carefully directed into sealed, isolated boxes Z^+ and Z^-. This operation is done with complete isolation from the environment, keeping their superposition state intact:

$$\begin{aligned}\Psi &= |\gamma\rangle \cdot |X^+\rangle_1 \cdot |X^+\rangle_2 \\ &\rightarrow |\gamma\rangle \cdot \frac{1}{\sqrt{2}}(|Z^+\rangle_1 + |Z^-\rangle_1) \cdot \frac{1}{\sqrt{2}}(|Z^+\rangle_2 + |Z^-\rangle_2). \end{aligned} \tag{2.15}$$

The boxes are transparent for the photon but opaque for the atoms. Atom 1's (2's) Z_1^+ (Z_2^-) box is positioned across the photon's v (u) path in such a way that the photon can pass through the box and interact with the atom inside in a 100% efficiency. Now let the photon go by BS_1:

$$\Psi = \frac{1}{\sqrt{2}^3}(i|u\rangle + |v\rangle) \cdot (|Z^+\rangle_1 + |Z^-\rangle_1) \cdot (|Z^+\rangle_2 + |Z^-\rangle_2). \tag{2.16}$$

After the photon was allowed to interact with the atoms, discard the cases in which absorption occurred

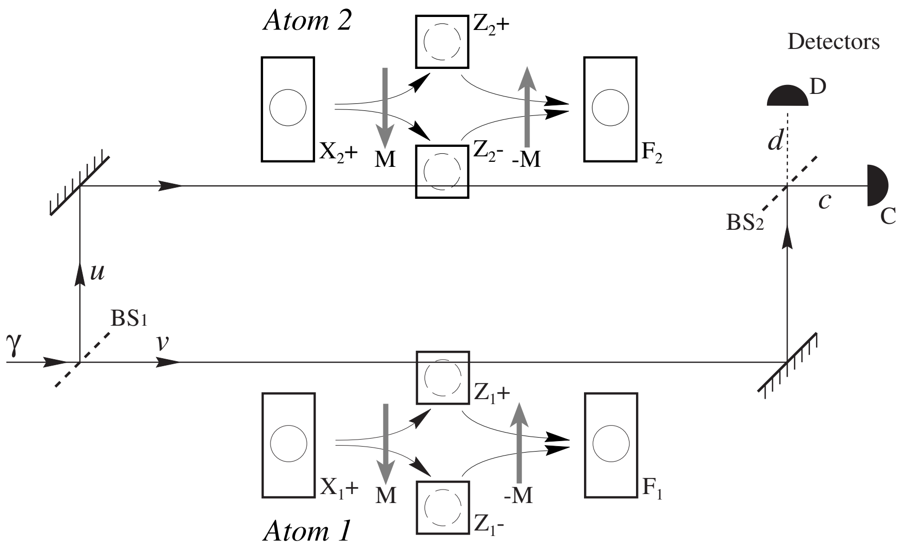


Figure 2.4: Hardy's experiment.

(50%), to get:

$$\Psi = \tfrac{1}{\sqrt{2}^3}(\quad i|u\rangle|Z^+\rangle_1|Z^+\rangle_2 + i|u\rangle|Z^-\rangle_1|Z^+\rangle_2 \\ +|v\rangle|Z^-\rangle_1|Z^+\rangle_2 + |v\rangle|Z^-\rangle_1|Z^-\rangle_2). \tag{2.17}$$

Now, let photon parts u and v pass through BS_2, following the evolution:

$$|v\rangle \xrightarrow{BS_2} \frac{1}{\sqrt{2}} \cdot (i|c\rangle + |d\rangle), \qquad |u\rangle \xrightarrow{BS_2} \frac{1}{\sqrt{2}} \cdot (|c\rangle + i|d\rangle),$$

giving:

$$\Psi = \tfrac{1}{4}(\quad |d\rangle|Z^-\rangle_1|Z^-\rangle_2 - |d\rangle|Z^+\rangle_1|Z^+\rangle_2 \\ +i|c\rangle|Z^-\rangle_1|Z^-\rangle_2 + i|c\rangle|Z^+\rangle_1|Z^+\rangle_2 + 2i|c\rangle|Z^-\rangle_1|Z^+\rangle_2). \tag{2.18}$$

If we now post-select only the experiments in which the photon was surely disrupted along its way, that is, take only the sub-ensemble where the photon hit detector D (as usual, in an uninterrupted MZI, the photon always reaches detector C):

$$\Psi = \frac{1}{4}|d\rangle(|Z^-\rangle_1|Z^-\rangle_2 - |Z^+\rangle_1|Z^+\rangle_2). \tag{2.19}$$

Consequently, the atoms, which have never met in the past, become entangled in an EPR-like relation. Unlike the ordinary EPR, where the two particles have interacted earlier, here the only common event in the past is the single photon wave function that has "visited" both of them.

Later on, in Section 6.4, I shall show how to achieve this result even without any common past event. Then, the measurement's effect on past evolution will become even more striking.

Chapter 3

Quantum Erasure

"I never forget a face, but in your case I'll be glad to make an exception."

Groucho Marx

"Quantum erasure" denotes an operation that constitutes time reversal of the measurement process, undoing the measurement's outcome and turning the wave function back to its initial state. It has become the focus of intensive study during the last few years, because of its far-reaching theoretical and technological consequenses, such as quantum computation and reversibility (see, for example [KSC94, BDS97]).

Quantum erasure occurs when a reduction of a quantum state is undone and the system returns to its original state. However, that is not an easy feat to do. It suffices for a single particle of the environment to interact with the system and "remember" its state to ruin any attempt for erasure. As a result, a quantum erasure is practically feasible only when a handful of particles, carefully isolated from the environment, interact with the system and measure it. In this case, it is possible to control and erase the measurement result for each of these particles. Once a macroscopic measurement device enters the scene, it is impossible to revert all of its composing particle and undo the measurement.

3.1 The Theory

Technically, measurement occurs when an object in a state of superposition

$$\Psi = \alpha|a\rangle + \beta|b\rangle, \tag{3.1}$$

interacts with a measurement device M. The measurement apparatus is prepared in the initial state M_0 and upon interaction with the measured object it undergoes the following transition:

$$M_0|a\rangle \longrightarrow M_a|a\rangle \tag{3.2}$$

$$M_0|b\rangle \longrightarrow M_b|b\rangle \tag{3.3}$$

The result is a measured object:

$$\Psi = M_0(\alpha|a\rangle + \beta|b\rangle) \xrightarrow{Measurement} \alpha M_a|a\rangle + \beta M_b|b\rangle. \tag{3.4}$$

Although the formalism shows that the measuring device enters a state of superposition with the measured object, this is the point where it clashes with every day experience. At this point also, the various interpretations of the quantum formalism differ. By interaction of an observer with the measurement system M, an apparent reduction of the state will occur and he or she will always find it either in the state $M_a|a\rangle$ or $M_b|b\rangle$.

Until now I used only the non-collapse formalism. At this point, if one could revert all the particles that recorded the measurement into their state prior to the measurement, M_0, or even to a different indistinguishable state M', the original state of superposition would be retrieved:[1]

$$\Psi = \alpha M_a|a\rangle + \beta M_b|b\rangle \xrightarrow{Erasure} M'(\alpha|a\rangle + \beta|b\rangle). \tag{3.5}$$

[1]Indeed, the possibility to perform such erasure experiments proves that in this stage no collapse occurs.

Note that if the measurement device makes any contact with the environment, it will be transformed from some initial state E_0 into a state that is related to the state of the measurement device (either E_a or E_b). Equation (3.4) becomes:

$$\Psi = E_0 M_0(\alpha|a\rangle + \beta|b\rangle) \xrightarrow{Measurement} \alpha E_a M_a|a\rangle + \beta E_b M_b|b\rangle, \tag{3.6}$$

and the erasure attempt will fail:

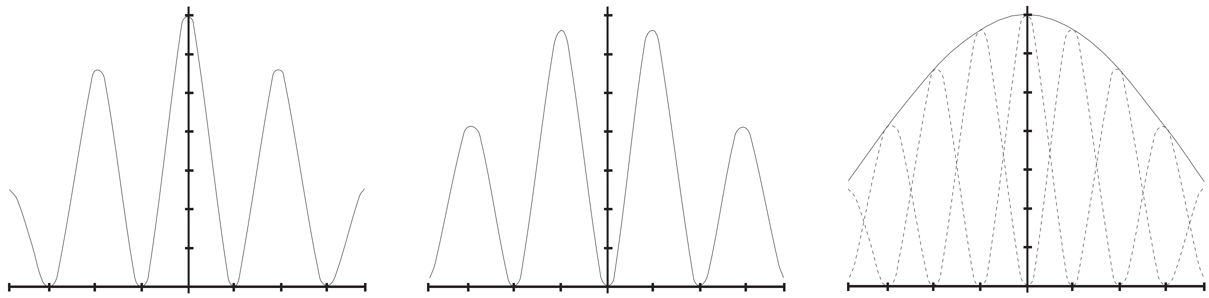
$$\Psi = \alpha E_a M_a|a\rangle + \beta E_b M_b|b\rangle \xrightarrow{Erasure} \alpha E_a M'|a\rangle + \beta E_b M'|b\rangle. \tag{3.7}$$

The two distinct configurations of the environment, E_a and E_b, are now entangled to the two states $|a\rangle$ and $|b\rangle$. This suffice to cause any future measurement to attach either to $|a\rangle$ or to $|b\rangle$. Therefore, in order to allow for quantum erasure, both the object and the measurement apparatus *must* be kept isolated from the environment, and any leak of information will result in a "collapse" of the state and the erasure will fail.

3.2 *Post Hoc* Erasure

Erasure might also be made *after* the measurement of the interference pattern. A notable work in this regard is due to **Marlan Scully** *et. al.*'s [SEW91], who demonstrated erasure of a quantum measurement in a double-slit experiment with atoms. In this experiment, the two parts of the atom's wave function, prior to being reunited, pass through a cavity where the atom emits a photon, telling which path did the atom traverse. Immediately after the emission, the photon is erased, thereby erasing also the "which path" information. Scully *et. al.* showed that after leaving the cavity, the two atom beams could be reunited, giving rise to an interference pattern.

However, the experiment could be modified in such a way that the taletelling photon is kept isolated while the atom beams are interfered. In this case, no interference pattern would be detected. Most interestingly, the which-path information could then be erased and the interference pattern retrieved.



(a) Interference due to ψ superposition.

(b) Interference due to ϕ superposition.

(c) The sum of ψ and ϕ shows no interference.

Figure 3.1: How can a *post hoc* measurement reveal interference pattern?

Scully *et. al.* showed that the resulting non-fringed interference pattern is in fact a superposition of two fringe patterns (see Figure 3.1), extracting one bit of information from the photon during the erasure process can discern between the two. That information can't disclose a which-path information for the atom, it can only identify whether it belongs to one fringe pattern or the other.

Formally, a particle (the atom) in a superposition $(|a\rangle + |b\rangle)$ is entangled to another particle (the photon, in the states $|\gamma_a\rangle$ and $|\gamma_b\rangle$, respectively) in a non separable manner:

$$|\Psi\rangle = \tfrac{1}{2}(|a\rangle|\gamma_a\rangle + |b\rangle|\gamma_b\rangle). \tag{3.8}$$

Since this state is not separable, no readable interference pattern will emerge if the two parts of the atom $|a\rangle$ and $|b\rangle$ are interfered. In what follows, I will use the following notation:

$$\begin{aligned} |\psi\rangle &= \tfrac{1}{\sqrt{2}}(|a\rangle + |b\rangle), \quad |\gamma_\psi\rangle = \tfrac{1}{\sqrt{2}}(|\gamma_a\rangle + |\gamma_b\rangle), \\ |\phi\rangle &= \tfrac{1}{\sqrt{2}}(|a\rangle - |b\rangle), \quad |\gamma_\phi\rangle = \tfrac{1}{\sqrt{2}}(|\gamma_a\rangle - |\gamma_b\rangle). \end{aligned} \tag{3.9}$$

Now, the state $|\Psi\rangle$ can be rewritten using the new basis:

$$|\Psi\rangle \quad = \quad \tfrac{1}{\sqrt{2}}(|\psi\rangle|\gamma_\psi\rangle + |\phi\rangle|\gamma_\phi\rangle). \tag{3.10}$$

That means that the pattern revealed when interfering $|a\rangle$ with $|b\rangle$ is actually a result of the interference pattern of $|\psi\rangle$ superimposed on the pattern of $|\phi\rangle$, as can be seen in Figure 3.1 (c). In order to differentiate between the two patterns, one should interfere the two photon parts $|\gamma_a\rangle$ and $|\gamma_b\rangle$, and

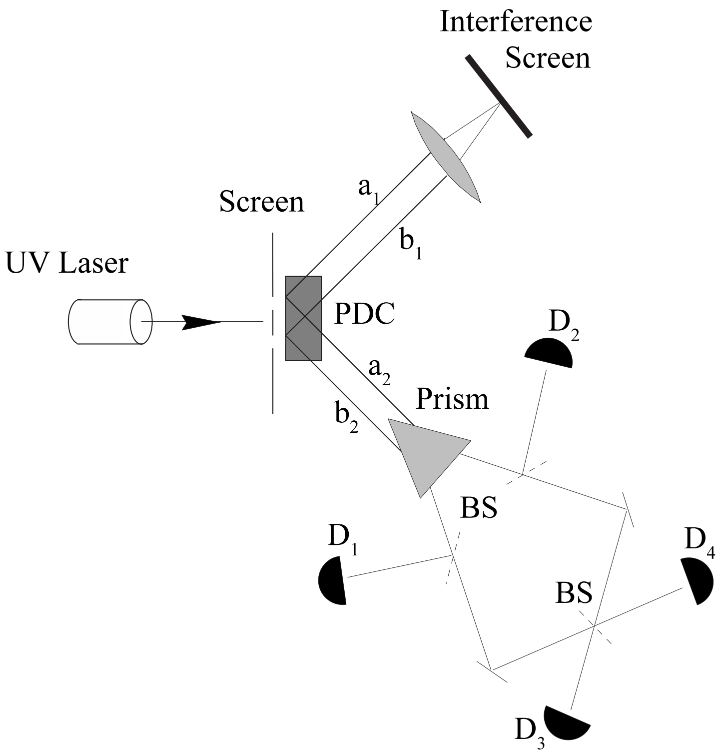


Figure 3.2: Kim *et. al.*'s quantum erasure experiment.

differentiate between the two orthogonal states $|\gamma_\psi\rangle$ and $|\gamma_\phi\rangle$. One can then tell apart $|\psi\rangle$ from $|\phi\rangle$ by, let's say, coloring the dots the atoms made on the interference screen: red for atoms that were coupled to $|\gamma_\psi\rangle$, and blue for atoms that were coupled to $|\gamma_\phi\rangle$. That will result reveal the two interference pattern superimposed on the screen. Note that the measurement of the photon can be performed even *after* the atom was hit the interference screen, since it only tells whether that particle should be assigned to the interference of $|\psi\rangle$ or $|\phi\rangle$.

A brilliant realization of this gedanken experiment was carried out by Kim *et. al.* [KYK+00] (Figure 3.2). In their measurement, Kim *et. al.* sent a UV laser photon through a double slit. Following the slits were a **Parametric Down Converter** (PDC) – a BaB_2O_2 crystal that has the ability to split impinging UV photons into two entangled red photons, each having half the energy of the original photon. The result was a single UV photon wave function split into four entangled beams: one "half" of the UV beam, originating from the upper slit, is split into two entangled red photons a_1 and a_2, while the second "half", originating from the lower slit, is split into b_1 and b_2. The UV laser is faint enough for only a single photon to traverse the apparatus at any given moment.

The two quarter photon beams a_1 and b_1 (which originated from different slits) are reunited to produce an interference pattern on a screen. At that time, though, the other two photon beams at a_2 and b_2 still fly along. They can now be either interfered or measured for "which-way" information. Detectors D_3 and D_4 measure the two interference states $|\psi\rangle_2$ and $|\phi\rangle_2$, while D_1 and D_2 measure which way did photon 2 go, that is: $|a\rangle_2$ or $|b\rangle_2$.

The result is quite surprising, though perfectly in line with the prediction of Quantum Mechanics:

1. The interference screen alone reveals no pattern of interference. That is no surprise since photon 2 carries along the which-way information, hence no interference can take place.

2. When considering only the subset of cases where detectors D_1 or D_2 clicked, thereby registering the which-way information, no interference is revealed either.

3. When taking into account only the subset of cases where detectors D_3 or D_4 clicked, thereby erasing the which-way information and differentiating between $|\psi\rangle_2$ and $|\phi\rangle_2$, interference pattern is revealed on the screen!

The appearance of interference pattern *after* the measurement itself is illustrated in Figure 3.1. All hits which coincided with detections at D_1 reveal the interference pattern depicted in (a), while measurements that coincide with D_2 are depicted in (b). However, before differentiating $|\psi\rangle$ from $|\phi\rangle$ the sum, depicted in (c), shows no interference.

In the following chapter a gedanken experiment is proposed, that incorporates quantum erasure, though of a different kind with some novel advantages.

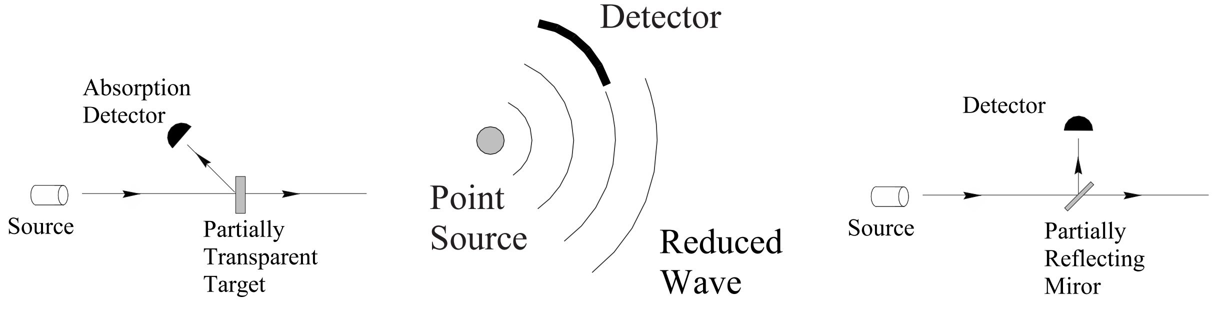
Part II

Novel Applications

Chapter 4

Partial Measurement

Whereas the previous part surveyed earlier works in quantum mechanics that provide the basis for this dissertation, this part presents the results of my own research. In this chapter the idea of partial measurement will be surveyed.



(a) Partial Transmitting Substance (b) Capturing part of the wave (c) Deflecting part of the wave

Figure 4.1: Partial Measurement.

4.1 Introduction

Nearly always, measurement is regarded as a single event, whereby the superposition of many possible states gives its place ("collapses") to one state. In reality, however, there can be many intermediate stages in the measurement process, stages that only change the eventual probabilities without yet giving a definite result [WZ79, BSCK92, KB92, DNR98]. In the following chapter I will discuss a procedure for partial measurement and its implications.

Partial measurement modifies the quantum state, turning a state of superposition not into a definite outcome but into a greater probability for one. Partial measurement occurs when only part of the wave function is transferred to the detector. Its roots lie at the end of the 1970's [WZ79, Bar80] when a two-slit like experiments were shown to extract partial knowledge about the particle's path in a price of partial blurring of the interference pattern.

Partial measurement can be achieved by several means (Figure 4.1):

1. Use, as a measurement device, a material having a small interaction cross-section (interaction probability) with the observed particles. When the particles' beam goes through such a material it absorbs part of the wave function. Sometimes, such absorption results in a full measurement leading to a reduction of the wave function. However, even if no particle was absorbed, the wave function suffered some kind of measurement, a partial one. Such a non-measurement, or **Null Measurement**, was explored by Kaloyerou and Brown [KB92].

2. Perform a full measurement, but only on a part of the region in which the wave function resides. This kind of experiment was proposed already at the 1950's by **Mauritius Renninger** [Ren53], much to Einstein's delight [Jam74, page 494]. As a matter of fact, every time we perform a null measurement on a photon that was emitted by a spherical source we perform such a partial measurement. In particular, every day we perform innumerable such a partial measurement on solar photons. Each photon's wave function is spread over a sphere 2 Astronomical Units in diameter (about 300 million

kilometers), while our retina covers a tiny fraction of that area. Each photon that is not captured by our retina is left with a “hole” in its wave function sphere, and the probabilities for its measurement in every other place rises a little.

When a measurement occurs, the whole wavefront immediately reduces to the measured value[1]. However, should the measurement fail to detect the particle, the wave function is altered and a “hole” is exposed in the area covered by the detector. A detector situated behind our detector (in regard to the wave direction) will never register a click. When proposed by Renninger, this idea was named the **Negative Result Experiment**.

For example, if a source emits a spherical photon waves, and two detectors are located $1m$ to the left and to the right of the source, each detector covers an area of eighth of the sphere area ($1.57m^2$). Assuming 100% detection efficiency, each detector has a probability of 12.5% to click for an emitted photon. However, if the right detector doesn't click, then it leaves a $1.57m^2$ area in the sphere where it is *known for sure* that the photon does not reside. The area over which the photon wave function is spread now is only 7/8 of the original area ($11m^2$), hence the probability for the left detector to click instantaneously changes from 12.5% to 14.3%. A question immediately arises: is this change in computed probability reflects only the change of our knowledge about the system, or it has some physical meaning? I will discuss this question later on.

3. One can use a partially silvered mirror to deflect part of the wave function for detection. This method is similar to the previous one, but it allows deflecting special parts of the wave function. For example, a **Calcite** crystal can distinguish between different polarization planes of light. Such a crystal can be used to deflect and measure a part with a certain polarization, thereby becoming a **Polarizing Beam Splitter** (PBS). In such a case, if no measurement was registered, then the resulting wave has a weaker component of polarization in the measured direction, hence the overall polarization plane of the wave rotates:

[1] This is a manifestly non-local operation that might leave detectable marks, see [AA81].

Assuming a photon with a 45° polarization, as it passes through a PBS it will be split into horizontal and vertical polarized components:

$$\phi = |\nearrow\rangle \longrightarrow \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\rightarrow\rangle). \quad (4.1)$$

Let a partial measurement be applied to 10% of the horizontal component by using a 10% silvered mirror on the path of the horizontal component. Such a measurement will result in a full measurement in 5% of the experiments – these cases will be post-selected out and hence ignored. In the remaining 95% a null measurement will ensue, transforming the quantum state:

$$\phi \longrightarrow \frac{1}{\sqrt{2}}(|\uparrow\rangle + \sqrt{0.9}|\rightarrow\rangle). \quad (4.2)$$

The polarization angle becomes:

$$\Theta = \tan^{-1}(1/\sqrt{0.9}) \approx 46.5°, \quad (4.3)$$

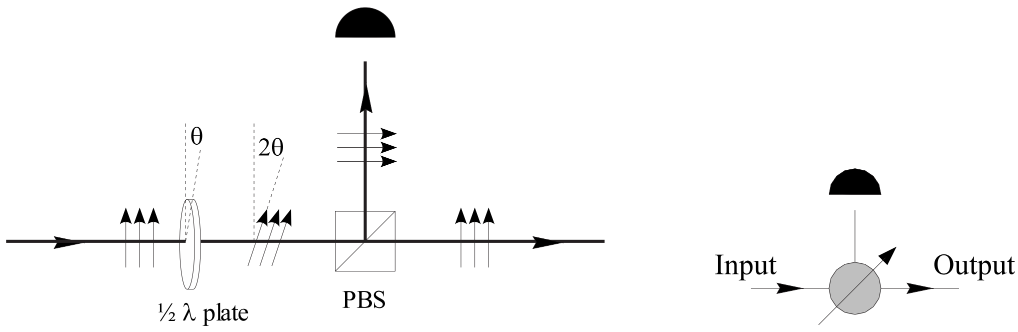
notice the rotation of the polarization plane from 45° to 46.5°.

Another possibility is to create a variable partial measurement apparatus (Figure 4.2 (a)). This device uses a half-wave-plate in order to rotate the polarization plane of a photon beam, and then a PBS to direct a part of the beam into a detector.

A half-wave-plate is an optical device that is constructed from a Calcite-like material. Such a material transmits horizontally and vertically polarized light in different speeds. By carefully choosing the thickness of a plate of such a material, it is possible to get a half-wavelength difference between the two components. Such a difference causes a rotation of the polarization plane: a beam that enters the plate with its polarization plane in an angle θ relative to the plate's vertical axis, will leave the plate rotated by an angle of 2θ.

Letting a beam of angle 2θ to pass through a PBS will divert the horizontal part of the beam and let the vertical part pass through. In our case, a vertically polarized beam will be transformed to:

$$|\Psi\rangle = |\uparrow\rangle \longrightarrow R_{2\theta}|\uparrow\rangle = \cos 2\theta|\uparrow\rangle + \sin 2\theta|\rightarrow\rangle, \quad (4.4)$$



(a) Apparatus schematics (b) Symbol

Figure 4.2: A Variable Partial Measurement

where $R_{2\theta}$ is the rotation operator.

The result is that there is a probability of $\sin^2 2\theta$ that the photon will be found on the horizontal polarization route and reach the detector, and a probability of $\cos^2 2\theta$ that no measurement will occur and the photon will leave the apparatus with a vertical polarization.

From here on, I will indicate partial measurements by the *intensity of the beam that is left unmeasured*. That means that in order to perform a partial measurement leaving a beam intensity of α, one has to rotate the half-wave-plate by:

$$\theta = \frac{1}{2} \cos^{-1} \sqrt{\alpha}. \tag{4.5}$$

In what follows, I will mark a variable partial measurement apparatus as in Figure 4.2 (b).

Harvey Brown *et. al.* harnessed partial measurement to present surprising properties of the wave function [BSCK92, KB92]. They showed that there is a difference between a quantum partial measurement and a stochastic, classical, one. A quantum partial measurement preserves the wave-like nature of the wave function, allowing for interference later on. While a classical measurement in a certain probability destroys the interference pattern. **Paul Kwiat** *et. al.* [KWH+95] used this principle in an experiment that harnessed the "**Quantum Zeno Effect**" [MS77] to improve the efficiency of Elitzur and Vaidman

“Bomb Testing” interaction free measurement [EV93]. These principles stresses once more that the quantum wave function has some kind of real, ontological, existence and is not only a representation of our knowledge about a quantum system.

Since partial measurement does not inflict a complete “collapse” on the wave function, I will use it in order to investigate various aspects of the measurement process. In this chapter I will explore the consequences of varying degrees of measurement and show that a partial measurement can undergo complete quantum erasure. Using EPR setting, I will show that every partial measurement nonlocally creates the same partial change in the distant particle, and that every erasure inflicts the same erasure on the distant particle. This allows an EPR experiment where the nonlocal effect does not vanish after a single measurement but keeps “traveling” back and forth between the two particles.

More specifically, I will study an experiment in which two distant particles are subjected to interferometry with a partial “which path” measurement. Such a measurement causes a variable amount of correlation between the particles. I will formulate a new inequality for same-angle polarizations, extending Bell’s inequality for different angles. The resulting nonlocality proof is highly visualizable, as it rests entirely on the interference effect. I shall also demonstrate how the measurement to be erased is fully observable, in contrast to prevailing erasure techniques where observation is forbidden and will inevitably destroy the erasure process.

As mentioned in section 1.2, Bell’s theorem [Bel64] has made it possible, for the first time, to experimentally demonstrate that quantum realism requires nonlocality. Later, the GHZ experiment [GHZ89] and Hardy’s [Har93a] proof without inequalities extended the proof to new domains. All these proofs, however, involve *complete* measurements. This is insufficient since, at the quantum level, measurement can be a continuous process, the intermediate stages of which have seldom been studied. In this chapter I will show similar proofs for partial measurements, as well as for the time-reversed process, namely, quantum erasure. Nonlocality will turn out to connect not only discrete events but continuous *processes* as well. The nonlocal influence will then appear to “bounce” back and forth, many times, between the distant particles during the measurements.

4.2 IFM and the Uncertainty Relations

Single-particle interferometry provides some of the most intriguing illustrations for quantum mechanical principles, and in recent years it has become technically feasible. Consider a photon entering a **Calcite** crystal positioned to divert the incident photon according to its polarization along the x axis (Figure 4.3). If the photon's polarization is 90° ($P_x = +1$, $|\uparrow\rangle$), it will be diverted to the lower path, whereas if it is 0° ($P_x = -1$, $|\rightarrow\rangle$) it will be diverted to the upper path. The calcite thus acts as a polarizing beam splitter (PBS), similar to the Stern-Gerlach magnet for spin $^1/_2$ particles. As long as no measurement has been made to find out which path the photon took – thereby leaving the photon's polarization undetermined – the photon will remain in a superposition of both paths and both polarizations:

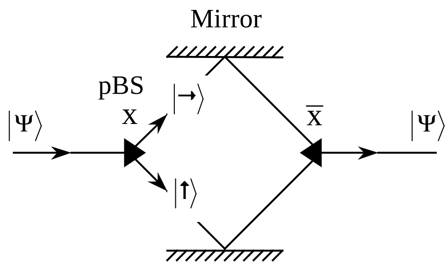


Figure 4.3: It is possible to restore photon's exact state as long as no measurement is performed.

$$|\Psi\rangle = c_1|\uparrow\rangle + c_2|\rightarrow\rangle. \tag{4.6}$$

So far, the splitting of the wave function is reversible. A second calcite, aligned at the x plane as the first but facing an opposite direction (denoted by $\bar{x}$), re-unites the resulting $|\uparrow\rangle$ and $|\rightarrow\rangle$ beams, so that the photon re-emerges in one single beam, in the same state as it has entered the first calcite (Figure 4.3).

The reversibility of splitting the photon along the x direction is demonstrated by the measurement of another, noncommuting variable. Before splitting, let the photon impinge on a calcite positioned in 45° to the x direction, henceforth named y polarization measurement. Suppose that the y polarization has been found to be +45°($P_y = +1$, $|\nearrow\rangle$). Then, let the photon split according to its x polarization and

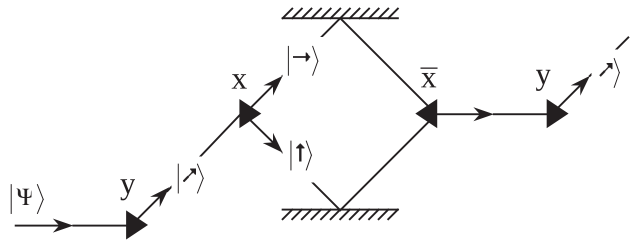


Figure 4.4: As long as no measurement is taken to tell the photon's polarization in the x direction, its polarization along the y direction remains intact.

re-unite again (Figure 4.4). If no measurement has been made between the splitting and re-uniting, we are left ignorant about the photon's x polarization. Consequently, the y polarization will remain intact: a final y measurement will *always* yield a polarization of $+45°$, just like the initial y measurement.

Suppose, however, that two detectors are placed on the two routes of the split wave function prior to its reunification (to enable the later reunification of the rays, let the measurements be of the non-demolition type, such that the detectors do not absorb the photon in case of detection). Since the $|\nearrow\rangle$ state is an even mixture of $|\rightarrow\rangle$ and $|\uparrow\rangle$,

$$|\nearrow\rangle = \tfrac{1}{\sqrt{2}}(|\uparrow\rangle + |\rightarrow\rangle), \tag{4.7}$$

in 50% of the cases the upper detector will click, and in the other 50% the lower one will. This measurement has an irreversible consequence: The knowledge we have gained about the photon's polarization will transform the quantum superposition into a classical mixture, represented by the density matrix:

$$\rho = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{4.8}$$

That takes the cost of blurring the y polarization, yielding a 50-50 distribution between $|\nearrow\rangle$ and $|\searrow\rangle$ at the last PBS. This is similar to the ordinary interference effect in a Mach-Zender Interferometer (MZI), where obtaining which-path information disrupts the photon's interference pattern.

Here a possibility for Interaction Free Measurement emerges (Figure 4.5): If, in order to measure P_x, we place only *one* detector on one of the two routes. Then, if the detector clicks the answer is clear: This

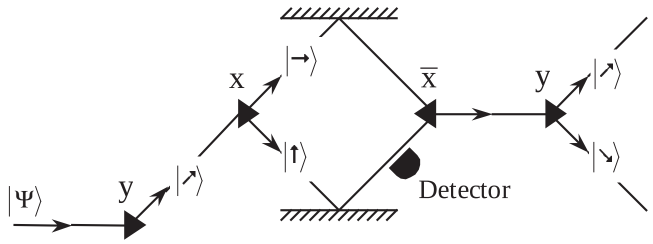


Figure 4.5: A measurement that discloses the photon's x polarization destroys the y polarization even when the measurement has been carried out by a single detector that did not click.

is an ordinary measurement, hence we should observe the above disruption of P_y. In the remaining 50% of the cases, when the single detector does not click, the photon's polarization is disclosed nonetheless and P_y is disrupted just because the silent detector *could* have clicked. As a result, there will be equal probabilities to measure that photon at the $+45°$ or the $-45°$ outputs of the last PBS!

IFM was reviewed in Chapter 2, in the present context it has two unique features that warrant attention: First, with a simple modification, IFM can be partial, leaving the wave function in a certain degree of superposition even after the measurement. Second, it can be completely reversed.

4.3 Introducing Partial Measurement

Into this apparatus one can introduce, instead of a detector, a partial measurement apparatus, see Figure 4.6. Here too, we can demonstrate the superposition of the photon's x polarization by performing two y polarization measurements, one before and one after the splitting and re-unification processes.

As long as no measurement is performed, the superposition will remain intact and all the photons will emerge in the same state as they entered the apparatus, *e.g.* $|\nearrow\rangle$.

Let us now apply a partial measurement of strength: $1-\alpha$ (where $\alpha < 1$). That means that out of the 50% of the cases where the photon could have been measured at the $|\uparrow\rangle$ state, only $1-\alpha$ will end up with actual measurement. These $\frac{1}{2}(1-\alpha)$ experiments will be ignored, since we are not interested in complete measurements. In the remaining $\frac{1}{2}\alpha$ experiments no detection will occur but the vertical polarized beam will be attenuated.

In the analysis below I will assume a partial measurement apparatus similar to the one depicted in Figure 4.2 (a). The vertical part will first be rotated by an angle of twice the plate's rotation angle $\theta = \frac{1}{2}\cos^{-1}\sqrt{\alpha}$, then a PBS will divert the horizontal part to a detector, leaving the vertical part untouched:

$$\begin{aligned}
|\Psi\rangle = |\nearrow\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\rightarrow\rangle) \xrightarrow{Rotation} \\
&\longrightarrow \frac{1}{\sqrt{2}}R_{2\theta}|\uparrow\rangle + \frac{1}{\sqrt{2}}|\rightarrow\rangle = \\
&= \underbrace{\frac{1}{\sqrt{2}}\sin 2\theta|\rightarrow\rangle}_{Measurement} + \underbrace{\frac{1}{\sqrt{2}}\cos 2\theta|\uparrow\rangle}_{Null\ Measurement} + \frac{1}{\sqrt{2}}|\rightarrow\rangle, \qquad (4.9)
\end{aligned}$$

post-selecting out the $\sin^2 2\theta = 1 - \alpha$ experiments which gave rise to a complete measurement:

$$= \sqrt{\alpha}\frac{1}{\sqrt{2}}|\uparrow\rangle + \frac{1}{\sqrt{2}}|\rightarrow\rangle + |\text{Measured Term}\rangle. \qquad (4.10)$$

Occasionally, I will use a normalizd notation:

$$\longrightarrow \sqrt{\frac{\alpha}{1+\alpha}}|\uparrow\rangle + \sqrt{\frac{1}{1+\alpha}}|\rightarrow\rangle. \qquad (4.11)$$

That is, increasing the IFM, slightly reduces the probability that the photon's polarization is $|\uparrow\rangle$, thereby increasing its probability to have a $|\rightarrow\rangle$ polarization. In what follows, I will denote such an operator as:

$$\text{Partial Polarization Measurement of Intensity } 1-\alpha \text{ in direction } \uparrow \equiv \hat{P}_{\uparrow\alpha}. \qquad (4.12)$$

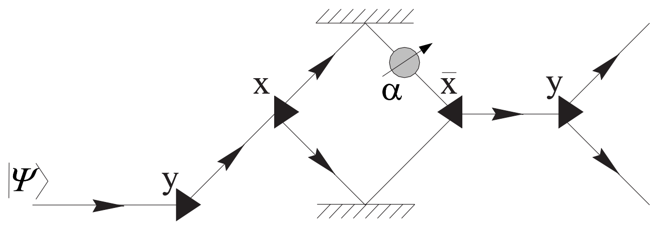


Figure 4.6: A variable measurement is applied to the polarized beams.

(Note again that α is the *unmeasured* intensity).

Formally:

$$\hat{P}_{\uparrow\alpha} \equiv \sqrt{\alpha}|\uparrow\rangle\langle\uparrow| + |\rightarrow\rangle\langle\rightarrow|, \tag{4.13}$$

marking

$$|\Psi_\alpha\rangle \equiv \hat{P}_{\uparrow\alpha}|\Psi\rangle. \tag{4.14}$$

This operator obeys the following multiplication law:

$$\hat{P}_{\uparrow\beta} \cdot \hat{P}_{\uparrow\alpha} = \hat{P}_{\uparrow\alpha\cdot\beta}, \tag{4.15}$$

that is, a partial measurement of $|\uparrow\rangle$ with intensity α followed by a partial measurement of the same beam with intensity β is equal to one partial measurement of the product intensity $\alpha\beta$.

The question of ontology *vs.* epistemology in quantum mechanics now poses itself: *Does partial measurement change merely our knowledge about the photon or is this a real physical change going on with the photon's state?* IFM provides a straightforward way to show that the latter is true.

Recall that, prior to the photon's splitting along the x direction, its polarization has been measured along the y direction and was found to be $|\nearrow\rangle$. All one has to do now is to reunite the $|\uparrow\rangle$ and the $|\rightarrow\rangle$ beams, and then measure again the y polarization. If no partial measurement was taken, the photon will emerge from the interferometer with its x polarization unmeasured, hence its y polarization will remain $|\nearrow\rangle$ with 100% certainty. If, however, an interaction-free partial measurement has been made, the y polarization will be only partly disrupted (see also [WZ79]). Instead of a pure $|\nearrow\rangle$ state, we will have $|\Psi_\alpha\rangle$ which is:

$$\begin{aligned} |\Psi_\alpha\rangle &= \hat{P}_{\uparrow\alpha}|\Psi\rangle + \\ &= \sqrt{\alpha}\tfrac{1}{\sqrt{2}}|\uparrow\rangle + \tfrac{1}{\sqrt{2}}|\rightarrow\rangle \\ &= \frac{1+\sqrt{\alpha}}{2}|\nearrow\rangle + \frac{1-\sqrt{\alpha}}{2}|\searrow\rangle. \end{aligned} \tag{4.16}$$

As the amount of measurement increases, y polarization gradually changes into a state of equal probabilities for $|\nearrow\rangle$ and $|\searrow\rangle$.

This change of the wave function is an objective, physical event. Measuring the photon's y polarization before and after the partial measurement will show that, at the statistical level, the y polarization has been disrupted monotonously with the knowledge we gained about the x polarization.

4.4 Partial Measurement exerts a Partial Effect on Non-commuting Variables

Let us now turn to the way partial measurement obeys the uncertainty principle. The effect of a partial measurement of P_x on P_y can be regarded as a rotation of the polarization plane (*cf.* Section 4.1): Drawing a vector with the $|{\rightarrow}\rangle$ component as the x ordinate and the $|{\uparrow}\rangle$ component as the y ordinate, will give $\theta_\alpha = \tan^{-1}\left(\frac{\langle \uparrow |\Psi_\alpha\rangle}{\langle \rightarrow |\Psi_\alpha\rangle}\right)$ as the angle of the polarization plane. The initial state of $|{\nearrow}\rangle$ has two equal components of the x polarization, namely, $|{\rightarrow}\rangle$ and $|{\uparrow}\rangle$, resulting in $\theta = 45°$. When a partial P_x measurement is taken, the $|{\uparrow}\rangle$ component diminishes causing the polarization plane to rotate clockwise, until a complete measurement gives a pure $|{\rightarrow}\rangle$ state (Figure 4.7).

We can now calculate the correlation coefficient between the initial and the final y polarizations as a function of the magnitude of the partial measurement (based on Equation (4.16)). The coefficient calculates the probability that the final state $|\Psi_\alpha\rangle$ will be equal to the initial state $|{\nearrow}\rangle$:

$$C_{y(\alpha)} = \left\| \langle \Psi_\alpha | \nearrow \rangle \right\|^2 = \left(\frac{1+\sqrt{\alpha}}{2}\right)^2. \tag{4.17}$$

$C_{y(\alpha)}$ ranges from 1 (when $\alpha = 1$, that is, no partial measurement) to 0.5 (when $\alpha = 0$, a full measurement took place and there is an equal probability to find $|{\nearrow}\rangle$ or $|{\searrow}\rangle$).

However, if we want to find the measurable effect that the partial measurement has at the output of the interferometer, we should check the polarization angle at the output. The deviation from the original

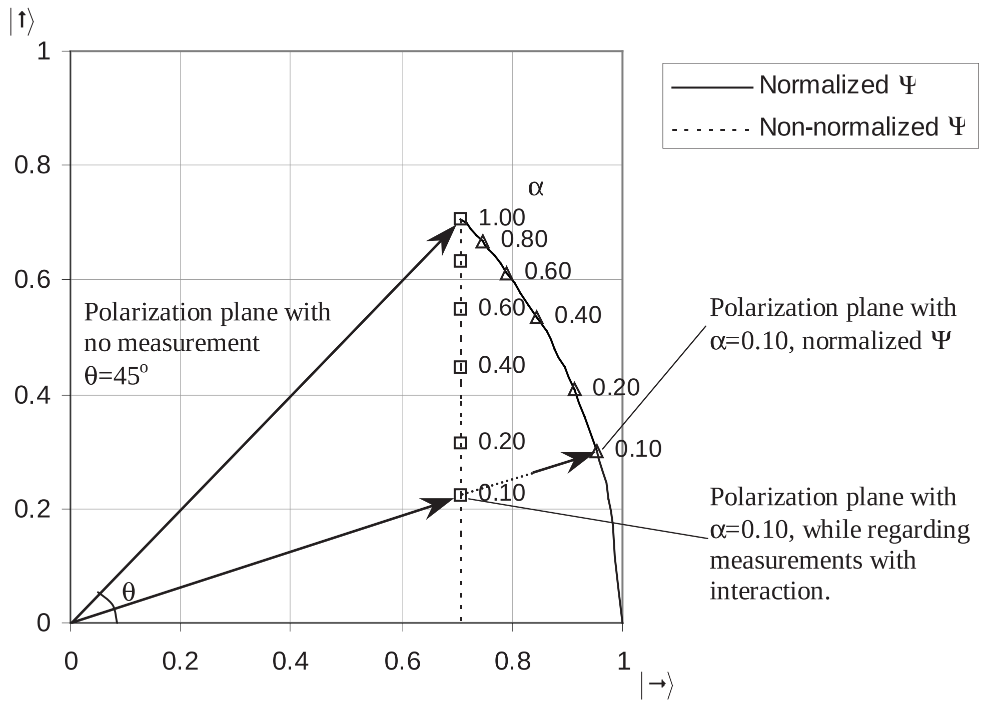


Figure 4.7: The polarization angle as a function of α.

state $|\nearrow\rangle$, where $\theta = 45°$, will have a measurable effect:

$$\begin{aligned} \theta_\alpha &= \tan^{-1}\Big(\frac{\langle\uparrow|\Psi_\alpha\rangle}{\langle\rightarrow|\Psi_\alpha\rangle}\Big) \\ &= \tan^{-1}\sqrt{\alpha}, \end{aligned} \tag{4.18}$$

For example, if partial measurements with intensity $\alpha = \frac{1}{2}$ have been carried, yielding no click, then the polarization angle at the output will be:

$$\theta_\alpha = \tan^{-1}\sqrt{\alpha} \simeq 0.61 \text{ rad}. \tag{4.19}$$

that means that if we measure the y polarization of the photons that reach the output, the probability to measure the original polarization ($|\nearrow\rangle$) at the output will be the square of the projection of a unit vector at an angel θ_α on a unit vector in the original direction (45°), that is, the square of the cosine of the angle difference:

$$P(f=|\nearrow\rangle)=\cos^2|\tfrac{\pi}{4}-\theta_\alpha|=\cos^2|\tfrac{\pi}{4}-\tan^{-1}\sqrt{\alpha}|\simeq 97\%. \tag{4.20}$$

4.5 Partial Measurement is Amenable to Complete Erasure

An intriguing peculiarity of partial measurement is that, in contrast to the ordinary one, it can sometimes be totally reversed. To do this, there is no need to time-reverse the operation of any detector; one can merely repeat the partial measurement on the photon's opposite branch (Figure 4.8).

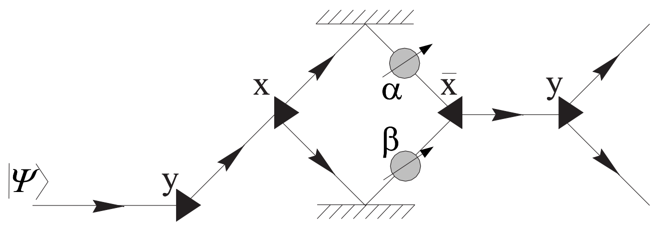


Figure 4.8: When the same amount of partial measurement is interaction-freely applied to both the $|{\rightarrow}\rangle$ and $|{\uparrow}\rangle$ branches, their overall effects cancel each other and the photon returns to the original superposition.

Let partial measurements by applied to both the upper and the lower path, both of intensity $1-\alpha$, thereby leaving each of the beams with intensity α. On average, in $1-2\alpha$ of the cases none of the detectors will click. This will completely undo both measurements and turn the wave-function back to the initial superposition:

$$\begin{aligned}|\Psi\rangle=\tfrac{1}{\sqrt{2}}(|{\uparrow}\rangle+|{\rightarrow}\rangle) \quad &\xrightarrow{Measurement} \quad \sqrt{\alpha}\tfrac{1}{2}|{\uparrow}\rangle+\tfrac{1}{2}|{\rightarrow}\rangle+|\mathrm{MT}\rangle\\ &\xrightarrow{Erasure} \quad \sqrt{\alpha}\tfrac{1}{2}|{\uparrow}\rangle+\sqrt{\alpha}\tfrac{1}{2}|{\rightarrow}\rangle+|\mathrm{MT}\rangle\\ &= \quad \sqrt{\alpha}\tfrac{1}{\sqrt{2}}(|{\uparrow}\rangle+|{\rightarrow}\rangle)+|\mathrm{MT}\rangle.\end{aligned} \tag{4.21}$$

According to the notation we used for the partial measurement, the erasure process will take the form:

$$\hat{P}_{\rightarrow\alpha}\cdot\hat{P}_{\uparrow\alpha}=\sqrt{\alpha}\hat{1}, \tag{4.22}$$

only diminishing the original state by $\sqrt{\alpha}$ (due to the cases in which a measurement will ensue).

This reversal occurs when the two partial measurements on the opposing branches are of the same magnitude. In the more general case, measuring $1-\alpha$ on the $|\uparrow\rangle$ branch and $1-\beta$ on the $|\rightarrow\rangle$ branch, the state then becomes:

$$\begin{aligned} |\Psi_{\alpha\beta}\rangle &\equiv \hat{P}_{\rightarrow\beta}\cdot\hat{P}_{\uparrow\alpha}|\Psi\rangle \\ &= \sqrt{\alpha}\tfrac{1}{\sqrt{2}}|\uparrow\rangle + \sqrt{\beta}\tfrac{1}{\sqrt{2}}|\rightarrow\rangle \\ &= \frac{\sqrt{\beta}+\sqrt{\alpha}}{2}|\nearrow\rangle + \frac{\sqrt{\beta}-\sqrt{\alpha}}{2}|\searrow\rangle, \end{aligned} \tag{4.23}$$

with correlation coefficient (as was defined in Equation (4.17)):

$$C_{y(\alpha\beta)} = \left\|\langle\Psi_{\alpha\beta}|\nearrow\rangle\right\|^2 = \left(\frac{\sqrt{\beta}+\sqrt{\alpha}}{2}\right)^2. \tag{4.24}$$

We now want to find the measurable effect that the partial measurement has at the output of the interferometer. For that purpose, we should check the polarization angle at the output. The deviation from the original state $|\nearrow\rangle$, where $\theta = 45°$, will have a measurable effect:

$$\begin{aligned} \theta_{\alpha\beta} &= \tan^{-1}\Big(\frac{\langle\uparrow|\Psi_{\alpha\beta}\rangle}{\langle\rightarrow|\Psi_{\alpha\beta}\rangle}\Big) \\ &= \tan^{-1}\frac{\sqrt{\alpha}}{\sqrt{\beta}} \\ &= \tan^{-1}\sqrt{k}, \end{aligned} \tag{4.25}$$

where k is the ratio between the beams' intensity: $k = \frac{\alpha}{\beta}$.

Notice that $\theta_{\alpha\beta}$ depends on the ratio k alone. That means, for example, that a measurement of 50% of the $|\uparrow\rangle$ branch ($k = \frac{1}{2}$) will yield exactly the same measurable results at the interferometer output[2], as a measurement of 90% of the $|\uparrow\rangle$ branch and 80% of the $|\rightarrow\rangle$ branch, or 99% of the $|\uparrow\rangle$ branch and

[2]Note that this effect is measured on the photons that *reach the interferometer output*, that is, the photons that are not captured by one of the two detectors *inside* the interferometer. The amount of photons reaching the interferometer output will decrease as α and/or β decreases – this effect is apparent in the formulation of $C_{y(\alpha\beta)}$, which diminishes with the decrease of α and β.

98% of the $|\rightarrow\rangle$ branch, etc. All give $k = \frac{1}{2}$. In other words, the exact measurement intensity on either branch does not matter, the only thing that counts is the ratio of the *unmeasured* intensities of the two branches!

In operator algebra:

$$\hat{P}_{\rightarrow\beta} \cdot \hat{P}_{\uparrow\alpha} = \hat{P}_{\uparrow\alpha/\beta}. \tag{4.26}$$

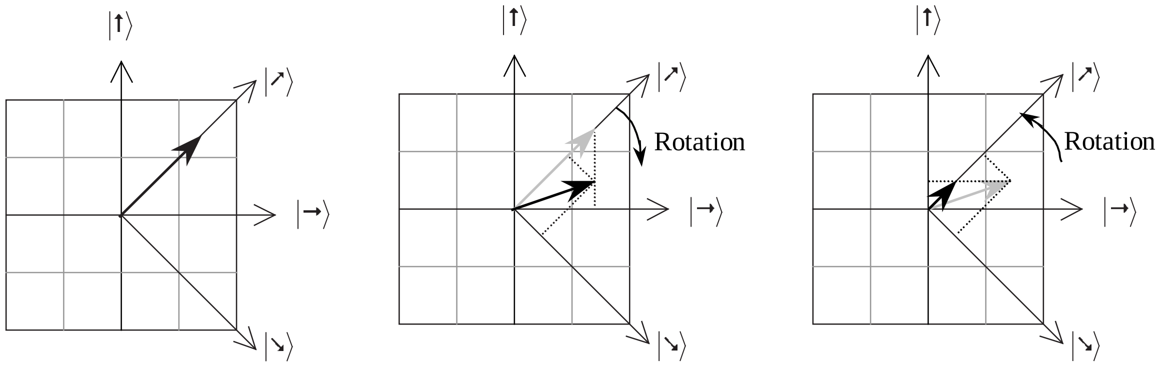
Consequently, the partial measurement operators in the x direction are commutating:

$$\hat{P}_{\rightarrow\beta} \cdot \hat{P}_{\uparrow\alpha} = \hat{P}_{\uparrow\alpha} \cdot \hat{P}_{\rightarrow\beta} = \hat{P}_{\uparrow\alpha/\beta} = \hat{P}_{\rightarrow\beta/\alpha}. \tag{4.27}$$

This ratio will later prove to be of crucial significance as a proof for nonlocal influence between distant photons.

An important feature of the erasure process is that it complies with the "no free lunch" principle. Any measurement on the $|\rightarrow\rangle$ path to restore the original $|\nearrow\rangle$ state will also diminish an equal part of the $|\uparrow\rangle$ part, as seen in Figure 4.9. This is evident by the fact that $C_{y(\alpha\beta)}$ decreases for some α where β is decreased from $\beta = 1$ to $\beta = \alpha$ (that is, from no measurement, to a complete erasure of the α measurement).

Until now we have seen that a partial measurement of the wave function causes a proportionate disruption of the interference effect. This setup also allows, in some cases, a complete erasure of the partial measurement and consequently a restoration of the interference pattern. In the next section we will see what can be achieved when a partial measurement is applied to entangled particles



(a) Initial setup: $|\Psi\rangle = |\nearrow\rangle$.

(b) A partial measurement on the $|\uparrow\rangle$ branch rotates the polarization plane: $|\nearrow\rangle$ diminishes while $|\searrow\rangle$ increases.

(c) Following a counter-measurement on the $|\rightarrow\rangle$ branch, the initial state is restored. The $|\searrow\rangle$ part is nullified but $|\nearrow\rangle$ diminishes too.

Figure 4.9: The cost of undoing a measurement.

4.6 Nonlocal Effects of Partial Measurement and Erasure, Revealed by Interferometry

Is it possible to show that such a partial measurement has nonlocal effects? The proof involves an experiment with two particles in a EPR-like Bell state. When both photons are subjected to interferometry, the partial measurement and erasure performed on each photon disrupt and restore, respectively, the interference effects of *both* photons.[3]

Consider, then, a pair of spacelike-separated photons, A and B, in the entangled Bell state $|\phi^+\rangle$, each entering an apparatus of the form described in Figure 4.8. This is a hybrid EPR-PM experiment (Figure 4.11). If P_x polarization will be measured for both photons, they will be 100% correlated but the P_y polarizations will lose correlation. Conversely, if no measurement is performed on the x polarization, their P_y correlations will remain intact.

Now let a partial measurement of intensity $1-\alpha$ be inflicted on the $|\uparrow\rangle$ branch of photon A. Photon

[3] For combining IFM and EPR experiments to prove the nonlocal nature of the former, see [Ryf92, Ryf99].

B will not undergo any measurement. A null measurement of photon A will perform an IFM, changing the wave function as in Equation (4.16). But here, due to the EPR state connecting the two photons, a unique state evolves. The partial measurement has partly disrupted the two photons' EPR entanglement:

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1|\uparrow\rangle_2 + |\rightarrow\rangle_1|\rightarrow\rangle_2\right)$$
$$\xrightarrow{Measurement} \sqrt{\alpha}\tfrac{1}{\sqrt{2}}|\uparrow\rangle_1|\uparrow\rangle_2 + \tfrac{1}{\sqrt{2}}|\rightarrow\rangle_1|\rightarrow\rangle_2 + |\mathrm{MT}\rangle. \tag{4.28}$$

This change causes a decrease in the correlation between their y polarizations:

$$\begin{aligned}
|\Psi_\alpha\rangle &= \hat{P}_{\uparrow_1\alpha}|\phi^+\rangle \\
&= \sqrt{\alpha}\tfrac{1}{\sqrt{2}}|\uparrow\rangle_1|\uparrow\rangle_2 + \tfrac{1}{\sqrt{2}}|\rightarrow\rangle_1|\rightarrow\rangle_2 \\
&= \frac{1+\sqrt{\alpha}}{2^{3/2}}\left(|\nearrow\rangle_1|\nearrow\rangle_2 + |\searrow\rangle_1|\searrow\rangle_2\right) \\
&\quad + \frac{1-\sqrt{\alpha}}{2^{3/2}}\left(|\nearrow\rangle_1|\searrow\rangle_2 + |\searrow\rangle_1|\nearrow\rangle_2\right) \\
&= \frac{1+\sqrt{\alpha}}{2}|\phi^+\rangle + \frac{1-\sqrt{\alpha}}{2}|\psi_y^+\rangle.
\end{aligned} \tag{4.29}$$

where $|\psi_y^+\rangle$ is the orthogonal "anti-EPR" Bell state in which the photons are entangled, but with an anti-correlated polarization:

$$|\psi_y^+\rangle = \frac{1}{\sqrt{2}}\left(|\nearrow\rangle_1|\searrow\rangle_2 + |\searrow\rangle_1|\nearrow\rangle_2\right). \tag{4.30}$$

note that $|\phi^+\rangle$ is a singlet state: in every direction it will give correlated polarization for the two atoms.

As we can see, the partial measurement did not break the entanglement between the two photons. Instead, the initial EPR state became "contaminated" by a certain amount of the anti-EPR state. This means that measurements of the y polarization of photons A and B have a probability of $\left(\frac{1+\sqrt{\alpha}}{2}\right)^2$ to yield correlated results, and a probability of $\left(\frac{1-\sqrt{\alpha}}{2}\right)^2$ to yield opposite results. The latter probability will grow as the magnitude of the partial measurements on P_x grows (that is, as α diminishes) until the correlation drops to the random level of 50-50 as α goes to 0 in case of a complete measurement (Figure 4.10).

The fact that the photons remain entangled, even *after* partial measurement was performed, will

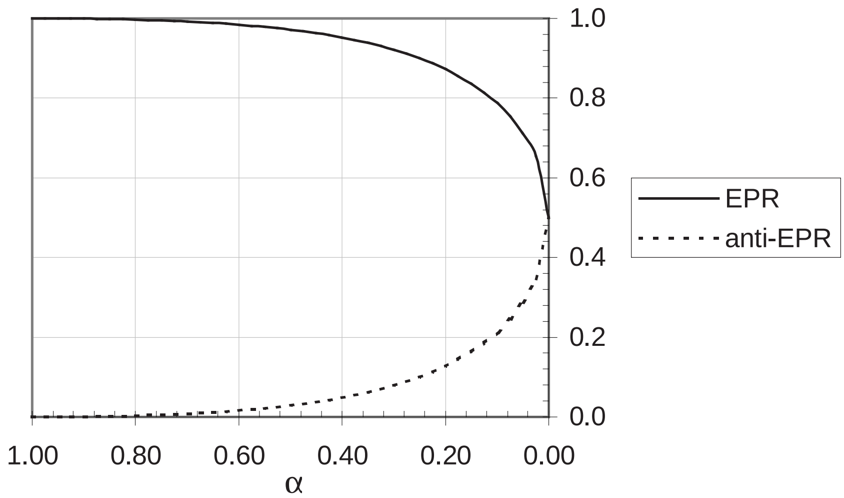


Figure 4.10: EPR (ϕ^+) and anti-EPR (ψ_y^+) parts as a function of α.

enables us, later on, to increase or decrease the amount of the anti-EPR component in subsequent measurements.

The next aim is to show that quantum erasure can undo not only the outcome of a measurement performed on the same photon, but also that of the *other* photon. Let us, first, extend the polarization notation introduced earlier to the EPR case as follows:

$$\text{Partial Measurement of intensity } \alpha \text{ on Particle } A \text{ in direction } \uparrow \equiv \hat{P}_{\uparrow_1 \alpha}. \tag{4.31}$$

Now, consider the erasure of a partial measurement as described in Section 4.5, applied to photon A of an EPR pair: A partial measurement of intensity $1-\alpha$ is applied to its $|\uparrow\rangle$ branch, while a $1-\beta$ partial measurement is applied to the $|\rightarrow\rangle$ branch. If the erasure works, *i.e.*, the measurement of both branches turns out to be interaction-free, the correlation between the two photons' y polarizations will be restored:

$$
\begin{aligned}
|\Psi_{\alpha\beta}\rangle &= \hat{P}_{\rightarrow_1\beta} \cdot \hat{P}_{\uparrow_1\alpha}|\phi^+\rangle \\
&= \sqrt{\alpha}\tfrac{1}{\sqrt{2}}|\uparrow\rangle_1|\uparrow\rangle_2 + \sqrt{\beta}\tfrac{1}{\sqrt{2}}|\rightarrow\rangle_1|\rightarrow\rangle_2 \\
&= \frac{\sqrt{\beta}+\sqrt{\alpha}}{2^{3/2}}\left(|\nearrow\rangle_1|\nearrow\rangle_2 + |\searrow\rangle_1|\searrow\rangle_2\right) \\
&\quad + \frac{\sqrt{\beta}-\sqrt{\alpha}}{2^{3/2}}\left(|\nearrow\rangle_1|\searrow\rangle_2 + |\searrow\rangle_1|\nearrow\rangle_2\right) \\
&= \frac{\sqrt{\beta}+\sqrt{\alpha}}{2}|\phi^+\rangle + \frac{\sqrt{\beta}-\sqrt{\alpha}}{2}|\psi_y^+\rangle. \qquad (4.32)
\end{aligned}
$$

It is now clear that when $\alpha = \beta$, the $|\psi_y^+\rangle$ part is zeroed out and the final state is the original $|\phi^+\rangle$ state (diminished by $\sqrt{\alpha}$ due to the experiments that will end up with a full measurement), restoring the initial entanglement of photons A and B.

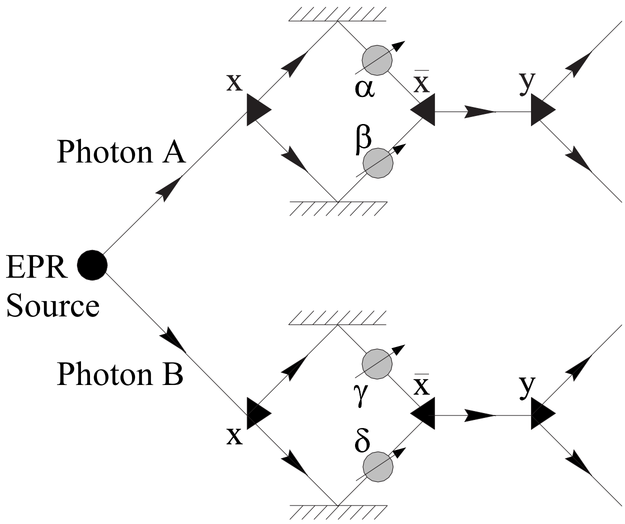


Figure 4.11: An EPR-interferometry experiment. As long as no measurement of the two photons' x polarizations is made, their final y polarization will be 100% correlated. When partial P_x measurements are carried out, the changes in the correlation between their y polarizations can demonstrate their nonlocal effects.

Note that the restoration process is reminiscent of a procedure proposed by Deutsch *et. al.* [DEJ+96] for "entanglement purification" of EPR-like pairs. However, their procedure is more general, hence suffers from lack of information about the partially-entangled state. Consequently, it necessitates destroying every other transmitted particle, which serves as "entanglement-control" particle, and then destroying also the accompanying particle, should the "entanglement-control" particle indicate a non-entangled state. The above experiment, in contrast, does not require "testing" the particles for entanglement. *Each and every* particle that is being "caught" by the detector in the "counter-measurement" is a non-entangled particle – that is, if the counter-measurement detector wasn't in place, the photon would have registered an anti-correlated polarization. Magically the quantum mechanical formalism ensures that only the non-entangled particles will be caught by the detector[4]!

4.7 Bearings on the Question of Local-Realism

The integration of partial measurement, quantum erasure, and EPR pairs allows one to farther examine the argument on local-realism in quantum mechanics. For when partial measurements and erasures are performed on two entangled photons, the local and nonlocal interpretations markedly differ[5]:

Local Argument A: Both the disruption of the correlation and its restoration are performed only by photon A's local interaction with the nearby detector, without affecting photon B whatsoever.

Nonlocal refutation: The correlation between the two y polarizations will be restored *even if we perform the partial x measurement on photon A and the undoing of this measurement on photon B.*

[4]This statement, of course, adopts a somewhat realist view of quantum mechanics, in which there is a Matter of Fact regarding the question whether a couple is correlated or anti-correlated even before the measurement. This point will be elaborated on Section 4.8.

[5]Proving nonlocal action is always difficult as adherents of locality often come up with very awkward yet not-impossible local mechanisms. Disproving such mechanisms is a tedious task, yet essential for a proof's completion. I therefore consider and disprove here all possible localist arguments.

Proof: Let's apply a partial measurement to photon B as well, denoting the intensities of the $|\rightarrow\rangle$ and $|\uparrow\rangle$ branches of photon B by γ and δ respectively (as per Figure 4.11). The pair's state will now be:

$$\begin{aligned} |\Psi_{\alpha\beta\gamma\delta}\rangle &= \hat{P}_{\rightarrow_2\delta} \cdot \hat{P}_{\uparrow_2\gamma} \cdot \hat{P}_{\rightarrow_1\beta} \cdot \hat{P}_{\uparrow_1\alpha}|\phi^+\rangle \\ &= \sqrt{\alpha\gamma}\tfrac{1}{\sqrt{2}}|\uparrow\rangle_1|\uparrow\rangle_2 + \sqrt{\beta\delta}\tfrac{1}{\sqrt{2}}|\rightarrow\rangle_1|\rightarrow\rangle_2 \\ &= \frac{\sqrt{\beta\delta}+\sqrt{\alpha\gamma}}{2}|\phi^+\rangle + \frac{\sqrt{\beta\delta}-\sqrt{\alpha\gamma}}{2}|\psi_y^+\rangle. \end{aligned} \tag{4.33}$$

The correlation between the initial and the final y polarizations would be:

$$\begin{aligned} C_{y(\alpha\beta\gamma\delta)} &= \|\langle\phi^+|\Psi_{\alpha\beta\gamma\delta}\rangle\|^2 \\ &= \left(\frac{\sqrt{\beta\delta}+\sqrt{\alpha\gamma}}{\sqrt{2}}\right)^2. \end{aligned} \tag{4.34}$$

The property similar to $\theta_{\alpha\beta}$ computed at (4.25) is the "angle", at the output, between the original state ϕ^+ and the orthogonal state ψ_y^+:

$$\begin{aligned} \theta_{\alpha\beta\gamma\delta} &= \tan^{-1}\Big(\frac{\langle\phi^+|\Psi_{\alpha\beta\gamma\delta}\rangle}{\langle\psi_y^+|\Psi_{\alpha\beta\gamma\delta}\rangle}\Big) \\ &= \tan^{-1}\frac{\sqrt{\beta\delta}+\sqrt{\alpha\gamma}}{\sqrt{\beta\delta}-\sqrt{\alpha\gamma}} \\ &= \tan^{-1}\frac{1+\sqrt{K}}{1-\sqrt{K}}, \end{aligned} \tag{4.35}$$

where K is the ratio between the intensities of the four measured beams:

$$K = \frac{\alpha\gamma}{\beta\delta}. \tag{4.36}$$

Note, that if we want to measure the probability that the photons will remain correlated in the $|\phi^+\rangle$ state after they leave the interferometers, we should perform a calculation similar to that of Equation (4.20), but the angle should be now compared to 90° and not 45°:

$$P(f = |\phi^+\rangle) = \cos^2|\tfrac{\pi}{2} - \theta_{\alpha\beta\gamma\delta}| = \cos^2\Big|\tfrac{\pi}{2} - \tan^{-1}\frac{1+\sqrt{K}}{1-\sqrt{K}}\Big|. \tag{4.37}$$

A few points are worth mentioning here:

1. The $|\phi^+\rangle$ component will always be greater than or equal to the $|\psi_y^+\rangle$ part. Hence, in the above setup, one can never reach a situation when the two photons are manifestly anti-correlated.

2. In the extreme case when either α, δ, β, or γ equals 0 (that is, a complete measurement was performed on one of the beams), the resultant state is an even blend of $|\phi^+\rangle$ and $|\psi_y^+\rangle$ states, $\theta_{\alpha\beta\gamma\delta} = \tan^{-1} 1 = \frac{\pi}{4}$. That means that, following a complete measurement of P_x on one branch, the correlation between the photons is completely destroyed.

3. The opposite extreme case occurs when $\alpha\gamma = \beta\delta$, thereby $\theta_{\alpha\beta\gamma\delta}$ asymptotically approaches 90°. Then, assuming all four partial measurements ended up interaction free, the two photons will restore the original $|\phi^+\rangle$ state of 100% entanglement. That is, even though α and β are different, a proper ratio of γ and δ *on the other photon* can erase the measurement and restore the initial correlations.

4. The process of "erasing" the measurement bears a cost: Getting rid of the $|\psi_y^+\rangle$ component will take the toll of diminishing the $|\phi^+\rangle$ part in an equal amount. That means that more measurements will end up with a click. If, for example, a measurement of 50% was taken on α, the result will be:

$$|\Psi_{1/2,1,1,1}\rangle = \frac{1+\sqrt{1/2}}{2}|\phi^+\rangle + \frac{1-\sqrt{1/2}}{2}|\psi_y^+\rangle, \tag{4.38}$$

whereas after a counter-measurement of 50% on β or δ, the result will be:

$$|\Psi_{1/2,1,1,1/2}\rangle = \frac{\sqrt{1/2}+\sqrt{1/2}}{2}|\phi^+\rangle + \frac{\sqrt{1/2}-\sqrt{1/2}}{2}|\psi_y^+\rangle = \frac{1}{2}|\phi^+\rangle. \tag{4.39}$$

Hence erasing the $|\psi_y^+\rangle$ part took the toll of yet another reduction of the $|\phi^+\rangle$ part by $\frac{1}{\sqrt{2}}$. That is the reason why we cannot retrieve the $|\phi^+\rangle$ state after a *complete* measurement on one of the branches: After such a measurement, the state will have equal amounts of $|\phi^+\rangle$ and $|\psi_y^+\rangle$: $\frac{1}{\sqrt{2}}|\phi^+\rangle + \frac{1}{\sqrt{2}}|\psi_y^+\rangle$. Trying to eliminate the $|\psi_y^+\rangle$ will take an equal toll on the $|\phi^+\rangle$ component, effectively nullifying it. That will leave only the measured term, that is, a fully measured photon: each and every photon will be caught by one of the intermediate detectors, and none will reach the output of the interferometer.

5. Since $\theta_{\alpha\beta\gamma\delta}$ depends on K alone, the process is *inherently* non-local. K is the ratio of the partial measurements on both particles and cannot be "compensated" on one particle without knowing the ratio of measurement on the other. This non-locality causes a Bell-like inequalities to break (see the refutation of Local Argument D in Section 4.9 for a demonstration of such an example).

Once, however, a pair of initially-entangled photons has survived the partial measurements of their $|{\rightarrow}\rangle$ and $|{\uparrow}\rangle$ branches with $K = 1$ – no matter whether the partial measurements were carried out on one photon or on both – they restore their entanglement, hence the correlation in their y polarizations. This offers a new extension of the EPR argument:

> *Just as quantum measurement imposes the measured polarization on the distant photon, so does quantum erasure obliterate the other photon's polarization.*

Equation (4.35) reveals another feature of the P_y correlation between the photons: The combined effects of partial measurements and partial "counter-measurements" do not comply with a simple addition or subtraction rules. If an interaction-free measurement has occurred, say, in 90% of the $|{\uparrow}\rangle$ branch, and in 95% of the $|{\rightarrow}\rangle$ branch, yielding a K value of $K = \frac{0.10}{0.05} = 2$, the resulting P_y correlation would be identical to that obtained by measuring just 50% of the $|{\rightarrow}\rangle$ branch alone (giving again, $K = \frac{1}{0.5} = 2$).

Now, when considering the interference effects of *two* such photons in the EPR setup, the nonlocality assumption yields another straightforward prediction that differs from the local assumption:

Local Argument B: The above deviation from ordinary subtraction rules stems from the interference effects occurring in each photon, regardless of what happens with the other photon.

Nonlocal refutation: The EPR state obliges the above subtraction rules to equally hold even when the $|{\uparrow}\rangle$ branch is measured in photon A and the $|{\rightarrow}\rangle$ branch is measured in photon B. This effect is obliged by Equation (4.37), which shows that the y polarization correlation at the interferometer output $P(f = |\phi^+\rangle)$ is a function of the measurement ratio K alone. Moreover, the restoration of the P_y correlation can be achieved by undoing operations on *both* photons at the same time – as long as K is kept equal to 1.

To summarize, in all cases in which interaction-free measurements are carried out on the opposing branches $|\uparrow\rangle$ and $|\rightarrow\rangle$, they mutually cancel out in the same way, *no matter whether they have been carried out on the same photon or on two entangled ones.* This indicates that each measurement affects the distant photon too.

4.8 Introducing the Experimenter's Free Choice

Let us consider the next difference between quantum theory and the local prediction:

Local Argument C: All the effects stem from a simple pre-established correlation between the photons, committing them to give the same results to the partial measurements.[6]

But such an argument enforces nonlocality in a new way:

Nonlocal refutation: In order for each single photon to be capable of responding to any strength of partial measurement (*à la* Figure 4.2), the photon must maintain a nonlocal connection between all the branches of its wave function traversing distant paths. In other words, if "erasure" detectors are placed, how can a "would be" non-correlated photon know that it should reach the branche with the detector, in order not to give a non correlated y measurement?

[6]Note that such a local account cannot be entirely classical. In a partial measurement apparatus suggested in Figure 4.2, one cannot assume that the photon traverses only one out of the N paths, because in that case no interference will be observed. The local account must therefore go along the lines of the "guide wave" interpretation, assuming the "empty waves", or the photon's wave function, traversed all the paths and not only the one taken by the corpuscle. Nonetheless, as I show below, this account does not restore locality either.

This aspect of the experiment parallels the last-minute choice of the polarization direction in Aspect and Grangier [AG86] experiment. A realization of the experiment would therefore require a random process for varying the amount of partial measurement.

Therefore, either the two photons maintain nonlocal correlation between them, or each photon maintains nonlocal correlation between its distant beams. The latter interpretation would join Hardy's [Har92b, Har94] and Albert *et. al.* [AAD85] proofs for the nonlocality of a single photon. Either way, nonlocality is inescapable.

The setup in Figure 4.11 also points out the difficulty in applying counterfactuals to quantum mechanics: Suppose we measure 50% of the $|\uparrow\rangle$ beam of photon A. A possible result of such an experiment might be that photon A finishes in the $|\nearrow\rangle$ path, while photon B goes on the $|\searrow\rangle$ path.

A possible description of such a situation will be: *There is a 3% probability (as per Equation (4.20)) that the photons will be anti-correlated, and this is one of these cases.* But then, a counterfactual can be presented: What would have been the result had we measured 50% of the $|\uparrow\rangle$ branch of photon B too? The answer is intriguing: Since doing so will completely undo the partial measurement on photon A, photon B must either fail the partial measurement (hit a detector) or completely agree with the y measurement of photon A. Since our photon did not agree with it, it must have failed the partial measurement!

This imposes another odd counterfactual:

> *When one places a counter-measuring partial measurement device on path B, they "magically" capture all the photons that* were about *to disagree with the y measurement of photon A, have we not placed the counter-measuring device.*

Such a teleological view is, of course, alien to physics, therefore pointing yet another price imposed by the realist view. One must rather accept the objective reality of the wave-function (as opposed to the "wave function is a description of our knowledge" approach). Only such a view can account for the fact that the partial measurement on photon B cancel exactly the 50% measurements done on photon A.

4.9 An Inequality for Partial Measurements

Let us now give a general nonlocality proof for partial measurements. We shall consider a local hypothesis that tries to maintain locality despite the above predictions and show that it must violate an inequality theorem.

Local Argument D: Each pair of photons uses a pre-established algorithm that assigns a definite P_y value for each partial measurement: For any magnitude of partial measurement that succeeded interaction-free, the photons would yield some preestablished y polarization. The resulting list of P_y values is infinite, matching every possible degree of partial measurement.

Nonlocal refutation: The alleged algorithm must satisfy two restrictions:

(1) In every pair, both photons must obey the same algorithm (though in reverse polarities): if photon A undergoes a measurement of 30% on its $|\uparrow\rangle$ branch, and photon B undergoes the same measurement of its $|\rightarrow\rangle$ branch, their P_y measurement *must* agree on each single experiment ($K = 1$, hence $\theta_{\alpha\beta\gamma\delta} = \frac{\pi}{2}$, and $P(f = |\phi^+\rangle) = 1$). This, indeed, explains the apparent "erasure," where opposite partial measurements on the two photons restore the initial correlations.

(2) However, the algorithm must assign the particular y polarization to the *ratio* of the intensities of the photon's $|\uparrow\rangle$ and $|\rightarrow\rangle$ branches (that is $\frac{\alpha}{\beta}$ for photon A and $\frac{\gamma}{\delta}$ for photon B). The correlation "angle" $\theta_{\alpha\beta\gamma\delta}$ makes this fact evident: A measurement of 50% on the $|\uparrow\rangle$ branch of photon A yields $\frac{\alpha}{\beta} = 0.5$, but many other measurements on photon B will result $\frac{\gamma}{\delta} = 0.5$ too (0%/50%, 60%/80%, 80%/90%, etc.), equating $\theta_{\alpha\beta\gamma\delta}$ to 1 and restoring the photons' P_y correlation.

Now, restrictions (1) and (2) refute the nonlocal argument by breaking a Bell-like inequality in the following way: Consider an experiment where photon A has $\frac{\alpha}{\beta} = 1.0$ and photon B has $\frac{\gamma}{\delta} = 0.5$. Here, $K = 0.5$, hence $\theta_{\alpha\beta\gamma\delta} = 1.4$, and $P(f = |\phi^+\rangle) = 0.97$, implying that the P_y correlation between the photons is disrupted in 3% of the cases.

If one believes in a pre-existing algorithm directing each photon, the following counterfactual must be true: Should B now have $\frac{\alpha}{\beta} = 0.5$ and it repeated the same measurement, but with a different opponent particle, C, with square the measurement ratio: $\frac{\gamma}{\delta} = 0.25$, the results *must* have been the same! Since B must give a P_y measurement according to it's $\frac{\alpha}{\beta} = 0.5$ ratio, it must measure exactly the same result that it gave for that ratio in the actual experiment (in accordance with restriction (1) – A and B must obey the same algorithm – and restriction (2) – the algorithm depends on $\frac{\alpha}{\beta}$ alone). Since K remains with the same value ($0.5/1 = 0.25/0.5 = 0.5$), $P(f = |\phi^+\rangle)$ remains 0.97, so B and C must give non-correlated results in 3% of the cases too.

This imposes another counterfactual: If A measured $\frac{\alpha}{\beta} = 1.0$ against C with $\frac{\gamma}{\delta} = 0.25$, they could give, at most, different results in $3\% + 3\% = 6\%$ of the cases (3% for the non-correlation between A and B, plus 3% non-correlation between B and C). However, when we compute $P(f = |\phi^+\rangle)$ for this case ($K = 0.25$), the result is 0.90. Which means that they *must* give non-correlated results in 10% of the cases! Since $10\% > 3\% + 3\%$, that condition cannot be met, and we conclude that the local argument is false. Q.E.D.

This proof will be generalized below for a broad range of ratios. That is, for a certain measurement ratio ρ, the disagreement between A and C is greater than the sum of disagreement between A and B plus B and C (see Figure 4.12):

If the difference between measurement ratios of particles A and B is $K = \rho$, the disagreement in P_y measurements will be:

$$\Delta_{A-B} = 1 - P(f = |\phi^+\rangle) = 1 - \cos^2\left|\frac{\pi}{2} - \tan^{-1}\frac{1+\sqrt{\rho}}{1-\sqrt{\rho}}\right|. \tag{4.40}$$

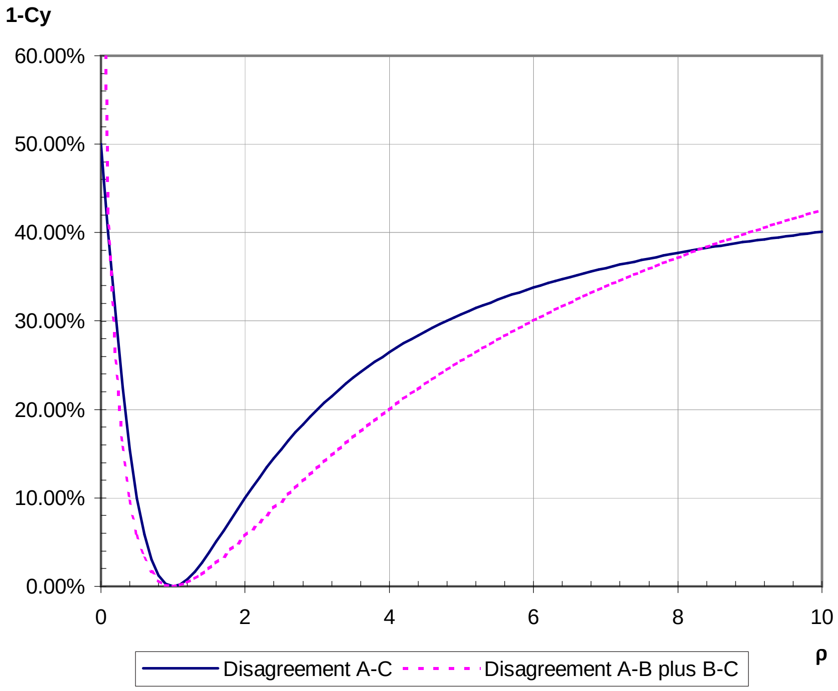


Figure 4.12: The disagreement between observers A and C is higher than the sum of disagreement between A and B plus B and C for a broad range of ratios, disproving locality by breaking a Bell-like inequality.

For simplicity, assume that the difference between measurement ratios of B and C will be ρ too, yielding the same value for Δ_{B-C} (since it depends on the ratio ρ alone), hence the maximal sum of the disagreement between A and C will be twice the above amount:

$$\Delta_{A-B} + \Delta_{B-C} = 2 - 2\cos^2\left|\frac{\pi}{2} - \tan^{-1}\frac{1+\sqrt{\rho}}{1-\sqrt{\rho}}\right|. \tag{4.41}$$

However, the measured disagreement between A and C will be according to $K = \rho^2$:

$$\Delta_{A-C} = 1 - \cos^2\left|\frac{\pi}{2} - \tan^{-1}\frac{1+\sqrt{\rho}}{1-\sqrt{\rho}}\right|. \tag{4.42}$$

In Figure 4.12 the graphs of these two functions are shown, which demonstrates that in the region $1 < \rho < 8.3$ the disagreement Δ_{A-C} is higher than the sum $\Delta_{A-B} + \Delta_{B-C}$, thereby disproving any possible local explanation.

It should also be pointed out that Local Argument D is especially ludicrous when we consider its *post hoc* explanations for the unique quantitative features of joint interferometry. Why should the ratio 50%-0% of measurement and erasure give the same result as 75%-50%, 90%-80% and so on? A local model can "explain" these phenomena only by adding arbitrary assumptions without any rationale other than the need to account for such unexpected results. In the nonlocal account, in contrast, these peculiarities are straightforwardly derived from *(i)* the very nature of interference, and *(ii)* the assumption that the two interferometries affect one another due to the quantum entanglement of the two photons.

Finally let the proof be extended to include all pairs:

Local Argument E: Perhaps only those photons that give rise to partial measurements maintain nonlocal correlation, while the others, which react to the measurement with a click in the detectors (complete measurement), have pre-fixed correlation and do not affect one other nonlocally. These photons, in other words, have "agreed" in advance to respond to the detectors with clicks, and therefore need not show correlation in their y polarization.

The refutation of this hypothesis (see also [EPR92]) is just like the proof presented in Section 4.8:

Nonlocal refutation: In order for some photons to be capable of responding to a certain number of detectors with a click, each such photon must maintain a nonlocal connection between all the distant parts of its wave function. For the photon cannot know in advance which of the paths is about to be measured and in which intensity. Therefore, *once the partial measurements confirm the nonlocal prediction, the complete measurements equally indicate nonlocal effects.*

4.10 The New Quantum Erasure and its Significance

Quantum erasure was discussed in Chapter 3 and an experiment was shown to implement such a procedure in Section 3.2. This experiment, however, has two shortcomings that obscure the uniqueness of quantum erasure. First, the experiment makes it impossible to know the actual result of the measurement that has been later erased. This impossibility is imposed by the very definition of the experiment: If an experimenter observes the result of a measurement, this observation itself becomes part of the measurement, hence erasure requires completely erasing that knowledge from the observer's brain as well. Therefore, one can only *infer* that detection and its erasure took place, but never know which beam gave rise to the initial click. The same holds for the proposals of Greenberger and YaSin, [GY89] Becker, [Bec98] and others, reviewed and refined by Kwiat *et. al.* [KSC94].

The proposed experiment overcomes this limitation. When the measurement is partial, it is as observable as any other measurement, and similarly its erasure. The reason for this has been pointed out earlier: Unlike the prevailing techniques, the proposed experiment rely on interaction-free measurement. Hence, it does not require the exquisite erasure of the measured information, and thereby avoids the enormous technical difficulties involved with proper thermodynamic isolation in current quantum erasure techniques. Somehow, quantum mechanics makes the would-be uncorrelated photons absorbed by the erasure detector, leaving only correlated photon to continue interaction-free out of the partial measurement apparatus.

Consequently, the only thermodynamic price one has to pay is that the closer the measurement gets to a complete measurement, the more likely it is to end up in a click, whereby erasure will be no more possible. Similarly, erasure itself might end up in a click. Still, in a large enough series of experiments, we can have many cases in which one can directly observe a measurement that is close enough to complete measurement, and then observe its complete erasure.

But the most intriguing consequence of quantum erasure is obscured by the second shortcoming of Scully *et. al.*'s experiment: They carried out the measurement and its erasure on *both* halves of the wave function. This, I suggest, is not only unnecessary but suppresses the nonlocal aspect of the process. However, this proposal suffers from the same shortcoming as the above erasure experiments: The measurement and its erasure can only be inferred and never directly observed. Brun and Barnett

[BB98], on the other hand, proposed to carry out the measurement on one arm and the cancellation on the other, arguing that both operations should affect both arms.

Unfortunately, all these experiments study single-particle interference, where the two parts of the wave function are eventually reunited. This does not allow nonlocality to be tested. A local theory could argue that both the measurement and its erasure affect only the measured region. An exception is Kim *et. al.*'s experiment [KYK$^+$00] which does use entangled photons, but suffers from the invisibility of the measurement and erasure processes.

The proposed partial measurement overcomes all these shortcomings. It allows a direct observation of both the measurement and its erasure. Furthermore, by performing the measurement on one photon and the erasure on the other, and by *not* uniting the two distant photons, one can positively affirm that *not only quantum measurement, but also its erasure, affects the other particle in the EPR experiment.*

4.11 Conceptual Implications

The above abundance of technical issues now allows for some simple yet illuminating philosophical conclusions.

In a discussion that has become a classic, Feynman [FLS65, volume III, page 1-1] defined the double-slit experiment as "the core of the mystery of quantum mechanics." In this chapter, I tried, in somewhat similar manner, to harness the familiar phenomenon of light polarization and interference to illustrate the fascinating features of IFM and nonlocality. It is this highly visualizable phenomenon upon which a proof for nonlocality is presented, not requiring Bell's theorem. In so doing, I have also extended the EPR argument with the conclusion that nonlocal effects are caused not only by a complete measurement but whenever the quantum state changes, either by partial measurement or by erasure.

Therefore, nonlocality needs not cease once the two particles in the EPR experiment are measured; they can remain entangled in spite of many successive measurements and erasures, maintaining the strange "dialogue" between them for a long time.

The experimental setup also extends Bell's proof in that, whereas Bell's inequality is based on the polarization measurements along varying angles, here, a new inequality was given that holds within the same angle of polarization for both particles.

Partial measurement gives a new twist to the question whether God plays dice. The particular experiment shows that not only does God cast a dice every time a polarization measurement is performed, but that, when the dice takes some time to fall, God preserves the right to change Her mind as many times as She pleases, rotating the polarization plane to and fro before a complete measurement discloses an unchangeable result.

To summarize, in this chapter fundamental peculiarities of quantum mechanics were revealed. Suppose that a classical object resides in one out of 10 closed boxes. Opening some boxes and not finding the object there only alters the observer's *knowledge* (or, better, ignorance) about the object's location. In quantum mechanics, in contrast, every such non-detection brings about a real change in the particle's state, detectable by measurements on a faraway EPR twin of the particle. It is this lack of differentiation between ontology and epistemology – any change in the observer's *knowledge* corresponding to precisely the same change in the *state* of the thing observed – that makes quantum mechanics so unique among all natural sciences.

Chapter 5

Non-sequential Behavior of the Wave Function

"How wonderful we have met with a paradox, now we have some hope of making progress"

Niels Bohr

Partial measurement, discussed in the previous chapter, is one efficient way to delay the measurement process in order to better study and comprehend it. In this chapter I'll present yet another novel technique of delaying measurement. Here is the basic idea: Rather than a macroscopic apparatus measuring the particle, let another particle do the measuring, *i.e.*, let an intermediating particle (or particles), in a delicate state of superposition, interact with the particle we want to measure. Measuring the intermediating particle will teach us about the state of the original one.

In the experiment reported below, the alleged progression of a photon's wave function is "measured" by a row of atoms, each in a state of a superposition. The photon's wave function affects only one out of the atoms, regardless of its position within the row. It also turns out that, out of n atoms, each one has a probability which is higher than the classical probability $1/n$ to be the single affected one. These results indicate that the wave function manifests not only non-local but also non-sequential characteristics.

5.1 Introduction

As demonstrated on Chapter 2, when a single photon goes through an MZI, its behavior indicates that it has somehow traversed both arms. However, when its position is measured during this passage, it turns out to have traversed only one arm. This is one of the notable manifestations of the measurement problem (see Section 1.1.10), for which several competing interpretations have been proposed (for sources containing a detailed review of the different interpretations, see note on page 3). For our present discussion, these can be crudely divided into two groups: "collapse" (*e.g.* Copenhagen, GRW) and "non-collapse" (*e.g.* Guide Wave, Many Worlds) interpretations.

At this point it should be noticed that both these groups seem to share one assumption. The photon – whether in the form of wave-plus-particle or of a wave function evenly spread over all available positions – is believed to proceed from the source to the detector sequentially through space-time. Hence, if a few objects are placed along its path, the photon is expected to interact with them one after another, according to the order of their positions, with the interaction events placed on the photon's null curve.

In this chapter an experiment is presented in which space-time sequentiality does not seem to hold.

5.2 Mutual IFM

Let us return to Section 2.1 and review the experiment proposed by Hardy. In Figure 5.1 a slightly skewed version of the same experiment is depicted.

A single photon traverses an MZI. In the normal course of events, due to interference, the photon will *always* leave the MZI hitting detector C. Within the MZI, the photon interacts with a spin $^1/_2$ atom

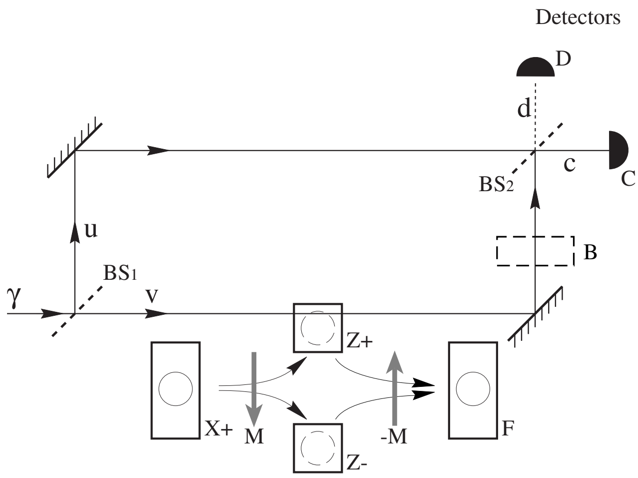


Figure 5.1: A photon in a Mach-Zehnder Interferometer interacting with a superposed atom.

which was prepared in a spin state $|X^+\rangle$, and then split into its two σ_z components $|Z^+\rangle$ and $|Z^-\rangle$ which are kept in separate boxes. The box containing the $|Z^+\rangle$ part is placed along arm v of the MZI.

The initial sate is:

$$\Psi = |\gamma\rangle \cdot \frac{1}{\sqrt{2}}(|Z^+\rangle + |Z^-\rangle). \tag{5.1}$$

After the photon is split by BS_1:

$$\Psi = \frac{1}{2}(i|u\rangle + |v\rangle) \cdot (|Z^+\rangle + |Z^-\rangle). \tag{5.2}$$

When the photon interacts with the atom, let us assume a 100% scattering probability, and when a photon is scattered by an atom it is detected in a 100% probability. In what follows, we will discard all cases of scattering (25%):

$$\begin{aligned} \Psi &= \frac{1}{2}(i|u\rangle|Z^+\rangle + i|u\rangle|Z^-\rangle + |v\rangle|Z^-\rangle) \\ &\quad +|scattering\rangle. \end{aligned} \tag{5.3}$$

Reuniting the photon by BS_2 gives:

$$\Psi = \frac{1}{\sqrt{2}^3} \cdot [i|c\rangle(|Z^+\rangle + 2|Z^-\rangle) - |d\rangle|Z^+\rangle]. \tag{5.4}$$

Now the atom's z boxes are joined, and the spin in the x direction measured, giving:

$$\begin{aligned}\Psi &= \frac{1}{4}|c\rangle \cdot (3|X^+\rangle - |X^-\rangle) \\ &\quad -\frac{1}{4}|d\rangle \cdot (|X^+\rangle + |X^-\rangle).\end{aligned} \tag{5.5}$$

Where it can happen that the photon hits detector D, while the atom is found in a final spin state of $|X^-\rangle$ rather than its initial state $|X^+\rangle$.

All that was done already on Section 2.1 and the implications on the interpretation of quantum mechanics were discussed. All these analyses, however, seem to assume space-time sequentiality. To show how this assumption can become strained, let us reconsider the above experiment with a slight yet crucial addition. Let a macroscopic object be placed after the atom on the v arm of the photon MZI ("B" on Figure 5.1). Here Equation (5.2) becomes:

$$\Psi = \frac{i}{2}|u\rangle \cdot (|Z^+\rangle + |Z^-\rangle) + |Scattering\rangle. \tag{5.6}$$

The scattering from the blocking object will be henceforth discarded, changing Equation (5.5) into:

$$\Psi = \frac{1}{2}(i|c\rangle - |d\rangle) \cdot |X^+\rangle. \tag{5.7}$$

The atom has retained its $|X^+\rangle$ state, indicating that the peculiar effect pointed out by Hardy can appear only if the two halves of the photon's wave function are allowed to reunite. In other words, the alleged "empty guide wave" or "collapsing wave function" will not exert their effect unless path v is allowed, *later*, to reach BS_2. Here, ordinary temporal notions are defied, and this defiance will become more prominent in what follows. I shall next point out a more peculiar effect of the wave function for which all the above interpretations, due to their sequentiality assumption, seem to be insufficient.

5.3 IFM with one photon and several atoms

Consider the setup given in Figure 5.2. Here too, one photon traverses the MZI, but now it interacts with three atoms, each in a state of superposition. In the general case, one can experiment with any number of atoms interacting in a row, but to display the peculiarity of quantum mechanics, three will suffice.

The main motivation to suggest such a setting is to try and differentiate between collapse and guide-wave interpretations. In a collapse interpretation the photon wave traverses the two arms of the MZI, it then reaches the first few atoms, interacting with them by impairing their superposition state. In a guide-wave interpretation, on the other hand, the guide wave must pass through all the atoms, hence harm the superposition state of all of them.

Formally, the initial state is a product of the photon and the three atoms:

$$\Psi = |\gamma\rangle |X_1^+\rangle |X_2^+\rangle |X_3^+\rangle. \tag{5.8}$$

After the photon's passage through BS_1 and the atoms' splitting according to σ_z:

$$\Psi = \frac{1}{4}(i|u\rangle + |v\rangle) \cdot (|Z_1^+\rangle + |Z_1^-\rangle) \cdot (|Z_2^+\rangle + |Z_2^-\rangle) \cdot (|Z_3^+\rangle + |Z_3^-\rangle). \tag{5.9}$$

Let us denote the initial state of the atoms:

$$|\phi\rangle = (|Z_1^+\rangle + |Z_1^-\rangle) \cdot (|Z_2^+\rangle + |Z_2^-\rangle) \cdot (|Z_3^+\rangle + |Z_3^-\rangle), \tag{5.10}$$

and the state of all atoms in their $|Z^-\rangle$ state:

$$|\psi^{\equiv}\rangle = |Z_1^-\rangle |Z_2^-\rangle |Z_3^-\rangle. \tag{5.11}$$

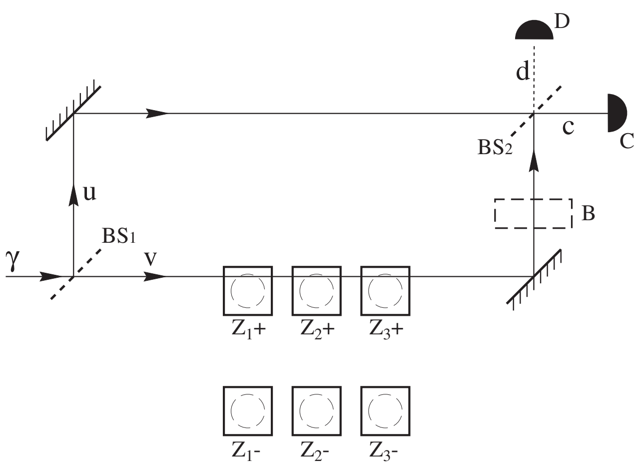


Figure 5.2: One photon MZI with several interacting atoms.

As in the previous experiment, we discard all the cases in which absorption occurs. Expanding Ψ will result in 16 terms with equal magnitude, 7 of which are of the form $|v\rangle|Z_i^+\rangle (i = 1, 2, 3)$, which will cause absorption of the photon and its detection. Hence, 44% $(= \frac{7}{16})$ of the experiments will end up with absorption:

$$\Psi = \frac{1}{4}(i|u\rangle \cdot |\phi\rangle + |v\rangle \cdot |\psi^{\equiv}\rangle) + |MT\rangle. \tag{5.12}$$

Note that the $|\phi\rangle$ part – where each atom has a 50/50 probability to be found in the Z^+ or the Z^- box – is coupled to the photon traversing the upper route u, while $|\psi^{\equiv}\rangle$ – where all three atoms are found in the Z^- state – is attached to the photon traversing the lower route v.

Now let us pass the photon through BS_2 and select only these cases in which it has lost its interference, hitting detector D:

$$\Psi = \frac{-1}{4\sqrt{2}} \cdot |d\rangle \cdot (|\phi\rangle - |\psi^{\equiv}\rangle). \tag{5.13}$$

Measuring the three atoms' spins now will yield, with a uniform probability, all possible results, *except* for the case where all the atoms are found in their $|Z^-\rangle$ boxes, which will never occur due to a destructive interference between $|\psi^{\equiv}\rangle$ and $|\phi\rangle$ (essentially, in this case the atoms will pose no obstacle to the traversing photon, hence it will retain its superposition state $(i|u\rangle + |v\rangle)$, and exit the interferometer towards detector C, the case we just post-selected out).

Reuniting the atoms' z boxes and measuring their σ_x will yield all possible combinations of $|X^+\rangle$ and $|X^-\rangle$ in uniform probability, except the case of all three atoms measuring X^+ which has a higher probability.

Let us, however, return to the stage before uniting the z boxes (as per Equation (5.13)). We know that at least one atom must be in the $|Z^+\rangle$ box to account for the loss of the photon's interference. Let us, then, measure one of the atoms' z spin. Without limiting the generality of the foregoing, let us measure atom 2's spin, and proceed only if it is found to be $|Z_2^+\rangle$ (57% of the cases):

$$\Psi = \frac{1}{4\sqrt{2}} \cdot |d\rangle \cdot (|Z_1^+\rangle + |Z_1^-\rangle) \cdot |Z_2^+\rangle \cdot (|Z_3^+\rangle + |Z_3^-\rangle). \tag{5.14}$$

Now unite the Z boxes of atoms 1 and 3 and apply the reverse magnetic field $-M$, and measure their spin in the x direction:

$$\Psi = \frac{1}{2\sqrt{2}} \cdot |d\rangle \cdot |X_1^+\rangle \cdot |Z_2^+\rangle \cdot |X_3^+\rangle. \tag{5.15}$$

Contrary to classical intuition, these atoms will *always* exhibit their original spin undisturbed, just as if no photon has ever interacted with them.

In other words, only one atom is affected by the photon in the way pointed out by Hardy, but that atom does not have to be the first one in the row, nor the last; it can be any one of the atoms. The other atoms, whose half wave functions intersected the MZI arm before or after that particular atom, remain unaffected.

We can prove, however, that although atoms 1 and 3 seem to be totally unaffected by the photon, *something* must have passed through the boxes. As in the previous section, let a macroscopic object be placed further along the v route, after the three atoms (object "B" on Figure 5.2). The above results will never show up. Here, all the atoms will give either Z^- (when the photon hits the obstacle), or remain in the original, X^+ state (when it does not). Hence, something must have passed through all three atoms, yet it has left the first and last unaffected.

Moreover, that "something" that seems to have passed through all the atoms must have done that at the precise moment. In order to dismiss any appeal for weird Feynman paths, note that the above formulation is compatible with the following setup: Let us place the atoms within sealed boxes, with apertures which open to the v path only for the minute interval during which the photon's wave function is supposed to pass through them. Now it is clear that no particle followed some convoluted path that went first through the middle atom, then the first and then the third.

The next result will deal the final blow on any realistic account in which a particular atom is affected by the photon at the moment of their interaction. We noted above that if we pick one atom, measure its position and find it in the Z^+ box, then that measurement will disentangle the two other atoms, causing their spins to reveal no trace of interaction with the photon. One might think that there is, prior to the measurements of the atoms, one particular atom that "has been" affected, and that the experimenter only has to be as lucky as to pick up that "right" atom that yields $|Z^+\rangle$. Not so: rather than the normal 33% probability to find the "right" atom, expected when there are 3 atoms, the probability is 56% for each atom:

Let us expand Equation (5.12) – which describes the state after post selecting out the events in which the photon was absorbed by the one of the atoms:

$$\begin{aligned}\Psi &= \frac{1}{4}\Big[i|u\rangle\Big(|Z_1^+\rangle|Z_2^+\rangle|Z_3^+\rangle + |Z_1^-\rangle|Z_2^+\rangle|Z_3^+\rangle + |Z_1^+\rangle|Z_2^-\rangle|Z_3^+\rangle \\ &\quad +|Z_1^+\rangle|Z_2^+\rangle|Z_3^-\rangle + |Z_1^-\rangle|Z_2^-\rangle|Z_3^+\rangle + |Z_1^-\rangle|Z_2^+\rangle|Z_3^-\rangle \\ &\quad +|Z_1^+\rangle|Z_2^-\rangle|Z_3^-\rangle + |Z_1^-\rangle|Z_2^-\rangle|Z_3^-\rangle\Big)+|v\rangle|Z_1^-\rangle|Z_2^-\rangle|Z_3^-\rangle\Big] + |MT\rangle. \end{aligned} \tag{5.16}$$

One can see that there are 9 terms with equal probability. However, for each atom i, there are 5 terms containing $|Z_i^+\rangle$, hence the probability to find the atom in the Z^+ box is $\frac{5}{9} = 56\%$.

In other words, every atom chosen by the experimenter, regardless of its position within the row, has a 56% probability to be "the only atom that has been affected by the photon." And once this atom gives this result, the other atoms will become nonlocally disentangled [1].

Note that the above analysis does not depend on the number of atoms or the index of the tested atom. For n atoms, the probability for any atom to be "the right atom" (that is, the only atom that has been affected by the photon) is $P = \frac{2^{(n-1)}}{2^n - 1}$ instead of the expected $P = 1/n$, and that: $\lim_{n\to\infty} \frac{2^{(n-1)}}{2^n-1} = 1/2$.

[1] No superluminal communication is entailed since a measurement on one atom cannot change the statistics of measurements on the other. Still, the correlation is Bell-like.

5.4 Analysis

From a formalist's point of view, it seems to be no surprise that the photon's v branch disappeared, leaving only part of the $|\phi\rangle$ state, where the unmeasured atoms remain undisturbed. This is because the photon's v branch was coupled to the term $|\psi^{\equiv}\rangle$ (all three atoms are in the Z^- state) which contains $|Z_2^-\rangle$. By post-selecting $|Z_2^+\rangle$ we exclude this branch altogether, leaving the others atoms superposed.

However, it is the attempt to reconstruct a comprehensible scenario from these correlations that gives a highly counterintuitive picture. For, if it is the measurement of the second atom that has cancelled the photon's v term, then, for the photon to reach that atom, it must have first passed through the first atom, and, later, through the third as well. If one tries to visualize this result obliged by the formalism, then, a single photon's wave function seems to "skip" the first atom that it encounters (remember that there can be any number of atoms there), then disturb the middle atom, and then leave the last atom undisturbed (and again, there can be several atoms there). Ordinary concepts of motion, which sometimes remain implicit within prevailing interpretations, are inadequate to explain this behavior.

The most prudent description of this result is that a wave function, when interacting with a row of other wave functions one after another, does not seem to comply with ordinary notion of causality, space and time.

This result is, in fact, one out of a family of peculiar results yielded by experiments of this kind, when a quantum mechanical object interacts with the measuring apparatus not directly but through another, intermediate quantum mechanical object. Another intriguing experiment of this family will be presented in the next chapter.

Chapter 6

Interfering with the Past

"Now it is fairly clear, if reality does not determine the measured value, then at least the measured value must determine reality"

Erwin Schrödinger [Sch35a]

The lesson of the previous chapter is that when one tries to infer, from the result of a measurement, about the chain of events in the quantum level, the outcomes are perplexing. In the following chapter the results will look even worse, that is, contradicting one another.

6.1 Introduction

As peculiar as quantum measurement is known to be, its strangeness is even greater when one tries to determine not merely the state of a system, but its entire *history*. Past events are supposed to be unchangeable, and as such the most essential aspect of reality. And yet, when a quantum measurement traces a certain history, it seems to take an active part in the very formation of that history. Note that this idea was explored in the early days of quantum mechanics in a paper by Einstein, Tolman, and

Podolsky [ETP31], but it seems like the implications of that paper were not appreciated enough at that time.

So far, however, this assertion has been merely philosophical. The most notable experiment supporting it, namely, the Einstein-Wheeler "delayed choice" experiment (discussed in the following section), is equally open to other, less radical interpretations. Could there be a more straightforward experiment, showing that the history observed is retroactively affected by observations carried out much later? In this article a few experiments of this type are discussed, and their implications considered.

6.2 The Delayed Choice Experiment

Let us begin with the "delayed choice" experiment [Whe78]. Discussing its limitations will later highlight the advantage of the proposed demonstration of "choosing history."

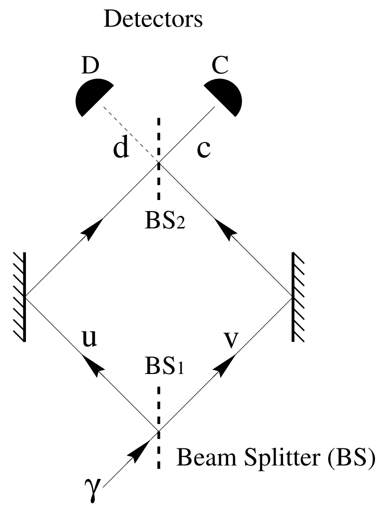


Figure 6.1: Mach-Zehnder Interferometer.

Let a Mach-Zehnder Interferometer (MZI) be large enough such that it takes light a long time to traverse it (Figure 6.1). Due to interference, every single photon traversing this MZI must exit hitting detector C (*cf.* Equation (2.4)). Suppose, however, that, at the last moment, the experimenter decides to pull out BS_2. In this case the photon hits either C or D with equal probability.

The most important point in this experiment is that the two options given to the experimenter's choice seem to entail two mutually exclusive histories. In the former case the photon seems to have been, *all along,* a wave that has traversed both MZI arms and then gave rise to interference. In the latter case the photon must have been – again, *all along* – a particle: if it has hit D it must have traversed only the right arm, and conversely for C. To make the result more impressive, Wheeler [Whe78] proposed to perform the experiment on photons coming from outer space, whereby the history thus "chosen" is millions-years long.

However, the delayed choice experiment is not scientific in the full sense of the word, as other explanations are possible within interpretations that do not invoke backward causation. One could, for example, just stick to the observed facts, refrain from any statement about the unobserved past and explain the experiment strictly in terms of wave mechanics or "collapse."

Can there be an experiment that indicates more strongly that past events are susceptible to the effect of future observation?

6.3 Interference between Independent Sources

Even more striking than the delayed-choice experiment is an effect that was still unknown to Einstein, namely, the interference of light coming from distant different sources. It was first proposed by **Robert Hanbury-Brown** and **Richard Twiss** [HBT57, HBT58] as a classical phenomena, and later demonstrated at the single-photon quantum level [PM67, Pau86] (Figure 6.2). It is odd that, although this experiment offends classical notions more than most other experiments known today, it has not yet received appropriate attention.

In a Pfleegor-Mandel-like experiment photons from distant, coherent, light sources interfere in a central region (Figure 6.2). An interference pattern will ensue, even when the radiation involved is of sufficiently low intensity that single photons reach the detectors one after another. In that case, each single photon seems to "have originated" from two distant sources.

Let us now examine two variations of this experiments that highlight its peculiar nature. First, it can have a delayed-choice variant: If the experimenter chooses at the last moment to pull out the BS, a click at detector C will indicate that a single photon has emerged in a corpuscular manner from only

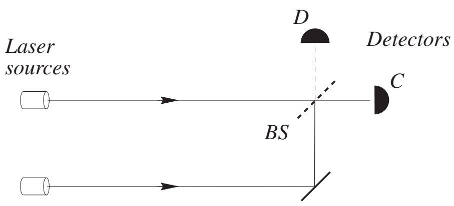


Figure 6.2: A schematic description of Pfleegor-Mandel experiment for interference between two distinct sources.

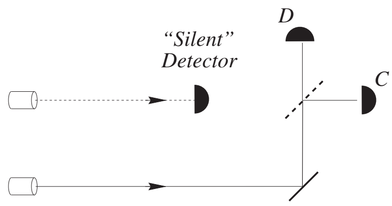


Figure 6.3: A variation of Pfleegor-Mandel experiment, implementing Interaction-Free Measurement.

one source, namely, the one facing the detector that clicked. If, on the other hand, she leaves the BS in its place, the interference will again indicate that the photon "has been emitted" by both sources in an undulant manner.

Next consider an IFM (Chapter 2) variant of this setting (Figure 6.3). Assuming that the phase between the sources is fixed for the time of the experiment, it can be arranged that all the photons will reach detector C. Now, if an object is placed next to one of the sources, it will occasionally absorb the photon. Therefore, when a photon eventually hits one of the detectors, it is obvious that it has been emitted only from the other, unblocked source. But then, in 50% of the cases, that photon will emerge from the BS towards the "dark" detector D, thereby indicating that, although it could have originated from only one source, it has somehow sensed the object blocking the other source!

How can two distant sources emit together a single photon? It is instructive to study this effect as a time-reversed version of the familiar case where a single photon is split by a BS and then goes to two distant detectors. In that case, there is an uncertainty as to which detector *will* absorb the photon. Similarly, in the above case, there is an uncertainty as to which source *has* emitted the photon.

This time-symmetry suggests constructing a new experiment. Consider first a simple case, where a single photon is split by a beam splitter, the result is a V-shaped setup (one source, two detectors). Such a split photon can entangle two unrelated particles so as to create an EPR pair. For example, a couple of two-levels atoms positioned across its two possible paths will become entangled due to the

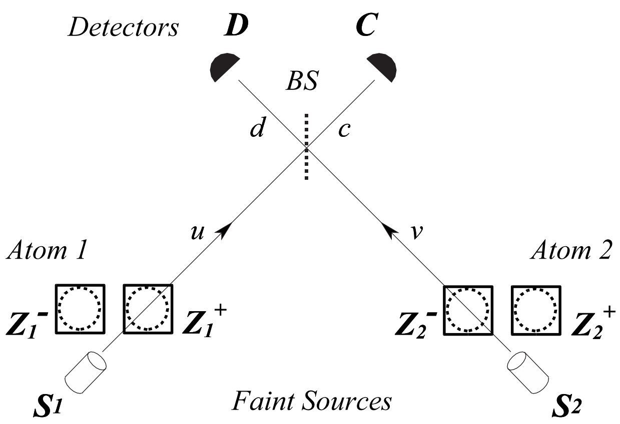


Figure 6.4: Entangling two atoms.

correlation between their ground and excited states. Can the more peculiar, Λ-shaped case (two sources, one detector) be similarly used to create an inverse EPR?

6.4 Inverse EPR ("RPE")

In order to achieve this peculiar result let us modify the experiment due to Hardy that was detailed on Section 2.2, only this time we shall harness the Pfleegor-Mandel effect to eliminate the common past of the two photon halves.

Let two coherent photon beams be emitted from two distant sources, S_1 and S_2, as in Figure 6.4. Let the sources be of sufficiently low intensity such that, on average, only one photon is emitted at a given time interval τ during which the photons traverses the apparatus. Let the beams be directed towards a distant BS. Two detectors are positioned next to the BS. In what follows the state of photon traversing the left arm (u) will be denoted by:

$$\phi_{\gamma u} = p|1\rangle_u + q|0\rangle_u, \tag{6.1}$$

while a photon traversing the right arm (v):

$$\phi_{\gamma v} = p|1\rangle_v + q|0\rangle_v. \tag{6.2}$$

where $|1\rangle$ denotes a "one photon" state (with probability p^2), and $|0\rangle$ denotes a "no photon" state (with probability q^2), $p \ll 1$, and $p^2 + q^2 = 1$.

Since the two sources' radiation is with equal wavelength, a static interference pattern will be manifested by different detection probabilities in each of the two detectors. Adjusting the lengths of the photons' paths v and u will modify these probabilities, allowing a state where one detector, D, is always silent due to destructive interference, while all the clicks occur at the other detector, C, due to constructive interference.

Notice that each single photon obeys these detection probabilities only if both paths u and v, coming from the two distant sources, are open. We shall also presume that the time during which the two sources remain coherent is long enough compared to the experiment's duration, hence we can assume the above phase relation to be fixed.

Next, let two spin $^1/_2$ atoms be prepared as in the previous chapter. The atoms are initially in a $\sigma_x = +1$ state, but are then split according to their Z spin. Each "half" is put into a separate box marked $|Z^+\rangle$ and $|Z^-\rangle$ respectively. One of each pair of boxes is placed acoss the photon's path, as indicated in Figure 6.4. The atoms' states are then:

$$\psi_{A1} = \frac{1}{\sqrt{2}}(i|Z^-\rangle_1 + |Z^+\rangle_1), \tag{6.3}$$

$$\psi_{A2} = \frac{1}{\sqrt{2}}(i|Z^-\rangle_2 + |Z^+\rangle_2). \tag{6.4}$$

After the photon was allowed to interact with the atoms, we discard the cases in which absorption occurred (50%), to get:

$$\Psi = \phi_{\gamma u} \cdot \phi_{\gamma v} \cdot \psi_{A1} \cdot \psi_{A2} \tag{6.5}$$

$$= \frac{1}{\sqrt{2}^3}(-i|u\rangle|Z^+\rangle_1|Z^+\rangle_2 - |u\rangle|Z^-\rangle_1|Z^+\rangle_2 \tag{6.6}$$

$$+i|v\rangle|Z^-\rangle_1|Z^+\rangle_2 + |v\rangle|Z^-\rangle_1|Z^-\rangle_2) \tag{6.7}$$

$$+|Scattering\rangle.$$

Now let the photons interfere over the BS, and post-select only the cases in which a single photon reached detector D, which means that one of its paths was surely disrupted, we get:

$$\Psi = \frac{1}{4}|d\rangle(|Z^+\rangle_1|Z^+\rangle_2 + |Z^-\rangle_1|Z^-\rangle_2), \tag{6.8}$$

which entangles the two atoms into a full-blown EPR state:

$$|Z^+\rangle_1|Z^+\rangle_2 + |Z^-\rangle_1|Z^-\rangle_2.$$

(Though this is an EPR-like Bell state, it is not the singlet state. However, it is possible to add a relative phase to the $|Z^+\rangle$ and $|Z^-\rangle$ boxes of the two distant parties, so as to reach the maximally entangled, singlet state.)

In other words, tests of Bell's inequality performed on the two atoms will show the same violations observed in the EPR case, indicating that the spin value of each atom depends on the choice of spin direction measured on the other atom, no matter how distant.

The two photon sources, though unrelated, must still be coherent in order to demonstrate interference. Dropping the coherency requirement would make the EPR inversion even more prominent. This has been accomplished by **Carlos Cabrillo** *et. al.* [CCGFZ99] in a different setup, devised for generating pairs of entangled atoms. Their setup involves atoms with three energy levels (Figure 6.5): two, mutually close "ground" states, $|0\rangle$ and $|1\rangle$, and one excited state $|2\rangle$. Two distant such atoms in $|0\rangle$ state are radiated by a weak laser beam tuned to the $|0\rangle \rightarrow |2\rangle$ transition energy. If a detector then detects a single photon of the $|2\rangle \rightarrow |1\rangle$ energy, and there is an uncertainty in regard to which atom emitted the photon, the atoms enter the entangled state $|1\rangle|2\rangle + |2\rangle|1\rangle$.

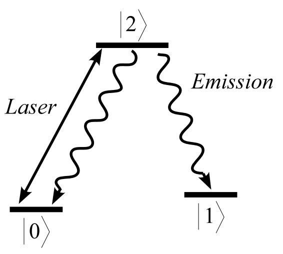


Figure 6.5: The energy levels in Cabrillo *et. al.*'s experiment.

Here, in the absence of coherence, one cannot talk about interference. Still, since only one photon is detected, the uncertainty about the photon's origin suffices to make the two atoms entangled, leading eventually to an EPR state.

Unlike the ordinary EPR generation, where the two particles have interacted earlier, here the only common event lies in the particles' future. These two versions, one involving coherent light and the other with incoherent light, highlight different peculiarities of the inverse EPR, henceforth termed "RPE."

6.5 Histories for Choice

The "RPE" experiment offers several options for studying the way in which measurement determines a history. Consider, first, its delayed-choice aspect:

- If the experimenter chooses at the last moment to pull out the BS, then the photon's two possible histories, *i.e.*, "it originated from the right atom" and "it originated from the left atom," become distinguishable. Consequently, the photon's "footprints" become distinguishable too and no entanglement between the atoms will be observed.
- Conversely, inserting the BS will entangle the two atoms, even though their interaction with the photon has taken place earlier. In other words, *what seems to be the generation of uncertainty only in the observer's mind, gives rise to a testable entanglement in reality.* Unlike the delayed-choice experiment, here the history "chosen" leaves observable footprints.

But, in addition to creating uncertainty at the end of the evolution, the coherent version (Figure 6.4) gives us the freedom to create uncertainty – or to dissolve it – also at the beginning of that evolution. For even after the photon was detected at D, one can perform two kinds of measurements on the atoms, measurements that will yield conflicting results:

- One can measure the position of each atom in one out of the two boxes. In this case, one atom must always be found in the intersecting box, while the other must always reside in the non-intersecting box. Consequently, there is only one possible history for the photon now: *It must have taken the* *path that was not blocked by the atom, never the other, blocked path.*

- On the other hand, one can unite the two boxes of each atom using an inverse magnetic field $-M$, and then measure the atom's spin along the X axis. Here, we give up the "which path" information about the photons, while every measurement will actually select an even mixture of the two possible histories.

All these variants are, in essence, erasure experiments (Chapter 3). When we insert the BS and reunite the atoms, we actually erase the still available information about the photon's two possible histories. Notice, however, that the present erasure experiments (*e.g.* [SEW91]) demonstrate only the negative result of this information loss, *i.e.*, the disappearance of the interference pattern. The RPE, in contrast, enables erasure to give rise to a positive result, namely, the entanglement of two distant atoms.

Another interesting feature of this experiment is the time in which the "fixation" of the history occurs. In contrast to the delayed choice experiment, in which the selection of the history must be made before the detection of the photon (actually, before the photon goes through the second BS), in the proposed experiment the differentiation between the histories can be made *after* the detection of the photon.

"*Nam et ipsa scientia potestas est* (for knowledge itself is power)" was an old maxim of the ancient Romans, but quantum mechanics rewards one for cases in which *ignorance* is generated.

6.6 Admit Backward Causation or Abandon Realism?

The time-symmetry of quantum theory's formalism is well known [ABL64] and has moreover become the cornerstone of some modern interpretations that render "affecting the past" the main characteristic of quantum interaction [Cra86, RA95]. As early as in 1983, Costa de Beauregard [dB83] gave a CPT-invariant formulation of the EPR setting that allows a time-reversed EPR. Can we apply such a formulation to the above case and assert that the late entangling event, *i.e.*, the detection of the photon, really affects backwards the two histories?

One might argue that the experiment does not really time-reverse the EPR setting because, in order to be sure that Bell's inequality will be violated, the atoms must be measured only after the detection of the entangling photon. Hence, the entangling event still remains in the past of the two correlated atoms. The EPR V shape, so goes the counter-argument, is thus merely flattened rather than turned upside down into a Λ shape.

Notice, however, that the entangling event can lie outside the past light cones of the two atoms' measurements. Here, the argument against backward causation must take the following form: "The two atoms begin to violate Bell-inequality only at the moment the photon was detected at D." This statement is relativistically meaningless. By bringing the entangling event itself into spacelike separation with the entangled particles, we actually render both the normal and inverse EPRs equally possible.

But what does "affecting the past" teach us about the nature of time? This question involves a deeper unresolved issue, that of time's apparent "passage." Adherents of the "Block Universe" model [Pri96], argue that time's passage is only an illusion. Consequently, all quantum mechanical experiments that seem to involve a last minute decision involve no free choice at all. For example, in the EPR, the experimenter's last-moment decision which spin direction to measure, or, in the "delayed choice" experiment, the last-moment decision whether to insert the BS or not, are "already" determined in the four-dimensional spacetime. Within this framework, RPE is just as possible as EPR.

The second alternative is that time has an objective "flow" [Pri80]. This requires some notion of "meta" time, in which our time flows. Then, the entangling effect could travel back and forth in that higher time dimension, inflicting a real retroactive influence once the "Now" has reached the entangling event.

Both views lie at present outside scientific investigation as both can be neither proved nor disproved.[1]

[1] However, it was shown elsewhere that Hawking's information erasure conjecture is more consistent with an objective time "passage." See [ED99]

Hence, a third and a much easier answer to the problem would be dismissing the entire issue by avoiding any reference to objective reality altogether, as in the Copenhagen Interpretation.

6.7 Inconsistency of the Past

Until now, the time evolution of the objects in question was consistent. In this section a gedanken experiment will be presented where the history, as is constructed backwards, at the end of the experiment, is not consistent. The experiment involves a twist in the EPR experiment. In Section 6.4 a time inverted version of the EPR experiment was presented, using the Pfleegor-Mandel effect (Figure 6.4). Two atoms are split, by a Stern-Gerlach magnetic field, according to their Z spin. Two photons travel, each through one half of the atoms. After the photons interfere at a central region and exhibit breakdown of their interference pattern (detected by the "forbidden" detector D), the two atoms become entangled in a full EPR state, with the intriguing feature that the two atoms share a common event not in the past but in the future.

Such a couple can now demonstrate Bell inequality violation, which rigorously proves that the two atoms indeed become maximally entangled into an EPR pair. It is therefore instructive to recall here Bell's inequality [Bel64] (Chapter 1.2), which supplies the standard nonlocality proof in the normal EPR case.

The theorem requires measurement of three observables, *e.g.*, spin measurements in the x, y, and z directions for a spin $^1/_2$ particle (see Figure 6.6). Let an ensemble of EPR pairs be created and let each atom of each pair undergo a measurement chosen randomly amongst the above three. Then let the incidence of correlations and anti-correlations be counted. By quantum mechanics, for maximally entangled pairs, all same-spin measurements will yield correlated results, while all different-spin pairs will yield 50%-50% correlated and anti-correlated results. And indeed, this is the result obtained by numerous experiments to this day ([AGR81a, ADR82, KMWZ95, ea98] to name a few). By Bell's proof,

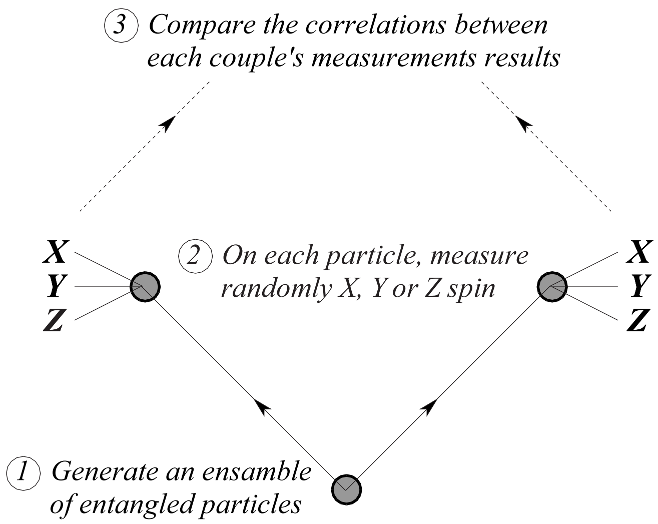


Figure 6.6: Proving that EPR pairs are maximally entangled: Maximally entangled pairs will always yield a correlated results for same-direction spin measurements, and 50%-50% correlated/anti-correlated for different-directions measurements.

such a result could not have been realistically pre-established. Hence, the spin value (up or down) of each particle is affected by the choice of spin direction (x, y, or z) measured on the *other*, timelike separated, particle, no matter how distant.

Let us now apply the above to the proposed experiment. When we spilt each atom so as to make it superposed in two boxes, it is split according to its Z spin. Then, at the end of the measurement, one can open the boxes and find out in which box the atom resides. This constitutes a z spin measurement. One can, on the other hand, reunite the two boxes using an inverse Stern-Gerlach magnetic field, then split the atom again in the x or y direction for a spin measurement. The rest of the experiment is just like that of the ordinary EPR: On an ensemble of inverse EPR pairs, spin measurements should be performed in a randomly selected directions. Having done that, quantum mechanics predicts that Bell's inequalities will be violated, thereby proving that the two atoms affect one another instantaneously, as in the ordinary EPR, even though their entangling event (the detection of the photon) occurs *after* the atoms' interaction with that photon (Figure 6.7).

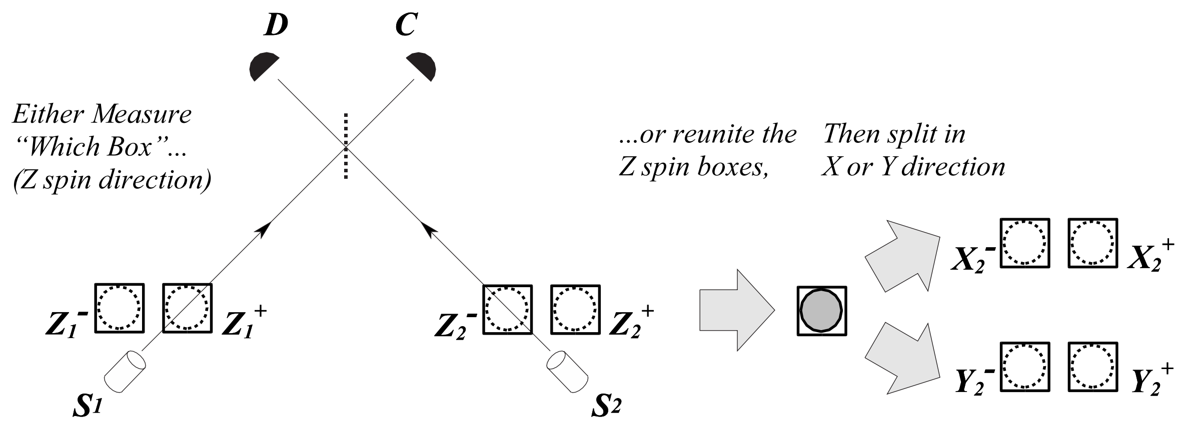


Figure 6.7: To test Bell's inequality on ensemble of pairs, measure each atom either in the z direction ("Which Box" measurement), or reunite the boxes and measure x or y directions.

It is here that a very peculiar situation emerges, unparalleled in other quantum paradoxes. In 55% (*i.e.*, $\frac{5}{9}$) of the cases (assuming random choice of measurement directions, as required above), at least one of the atoms will be subjected to a z spin measurement – namely, checking whether it resides in the intersecting or in the non-intersecting box (Figure 6.8). Suppose, then, that the atom is found to be in the intersecting box. This seems to indicate that *no photon has ever passed through this route, since it is blocked by the atom.* Consequently, the photon detected at detector D must have been emitted by the other source and the other atom must be in the non-intersecting box. This, indeed, will always turn out to be the case if one performs a "which box" measurement on the other atom.

Since the EPR state is a maximally entangled state, each and every pair in the ensemble must be entangled (see Elitzur *et. al.* [EPR92]). According to Bell's theorem, a strong contextuality takes place between the two atoms, and the results of any measurement taken on one of them depends on the kind of measurements applied (in any timelike event) to the other. However, for these 55% of the pairs in the ensemble one can portray a perfectly local history: One atom was in the crossing z box, the other in the non-crossing z box, one photon source emitted a photon which arrived to the BS and went to the "forbidden" detector. Here there could be no interaction between the atoms *even in the future*, nonetheless the atoms must have become entangled. That means that the two following assertions are

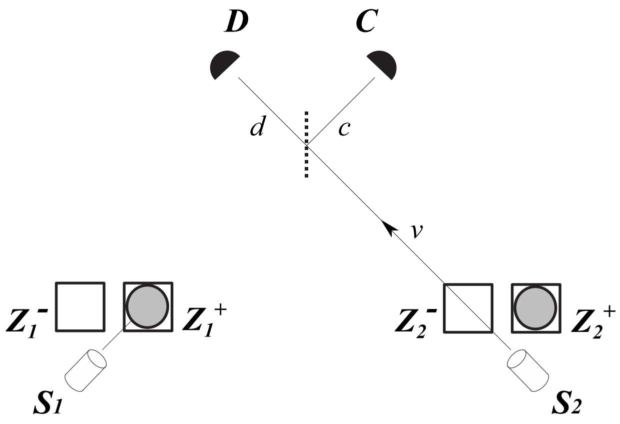


Figure 6.8: The situation after a "Which Box" measurement.

true, although they contradict each other:

- One atom was found in the intersecting box, blocking the entangling photon. Hence no interaction could have taken place between the two atoms whatsoever, and no dependency could appear between them.

- Before the measurement, the two atoms were in an EPR state. Consequently, Bell's theorem predicts that one atom's state is affected by the measurement performed on the other.

This case looks unique. As intriguing as the ordinary EPR is, it necessitates a causal connection between the two particles – they must, after all, be earlier emitted from the same source or interact in some other way. In the time-reversed EPR there is, again, a causal connection between the two atoms, namely, the future detection of a single photon that has somehow interacted with both of them. True, this is a bizarre causal connection because it lies in the future, but it is a causal connection nonetheless. The situation is much stranger with the present sub-ensemble of pairs in which a "which box" measurements (z spin) clearly singles out a local history for the pair. The entanglement between the two atoms in *this* case seems to be devoid of any causal connection between the atoms.

On the other hand, the result of the "which box" measurement that seems to indicate an absence of causal connection between the atoms is in itself due to this very causal connection. This seems to be a paradox of a new type. It does not stem from a contradiction between quantum mechanics and relativity, as in the EPR, nor from a contradiction between quantum and classical mechanics, as in Schrödinger's cat. Rather, it seems here that a quantum evolution can contain contradictory indications within itself, the result being logically equivalent to the statement: "This sentence has never been written."

This result accords with the results obtained in Chapters 4 and 5, as well as by Aharonov *et. al.* [AV90, RAPV95] in which the evolution of a quantum system does not seem to be self-consistent. As with **John Cramer**'s absorber theory [Cra86], Aharonov proposed to view any quantum interaction as the sum of two wave functions, one going forward in time from the source to the absorber/detector, while the other traverses the same spacetime path backwards. One of the most striking outcomes of this formalism was a group of highly counterintuitive predictions such as a *negative* number of particle traversing an MZI route [AMP+96]. It is no coincidence that the results reported in this thesis have been obtained for states of particles between successive measurements – just the place where Aharonov *et. al.* find their peculiar results.

I believe the explanation of these results may lie in a new interpretation of time's nature at the quantum level. But regardless of the interpretation, the effects themselves are intriguing enough to warrant further research.

Part III

Discussion and conclusions

Chapter 7

Summary

In this dissertation I have presented some intriguing gedanken experiments that emphasize the weirdness of the quantum world. More specifically, I have studied the measurement problem with a new approach based on delaying the measurement process in order to gain a few more insights into the nature of quantum phenomena. The experiments proposed are still all gedanken experiments, but, like erasure and IFM experiments during the last decade, they are worth being carried out. Moreover, these experiments by no means exhaust the new method's capability of producing surprising results; further experiments are conceivable.

In Chapter 4 partial measurement was presented. It allowed us to observe the tangible effect that interaction-free measurement inflicts on quantum systems. The partial measurement also allowed us to completely erase the effect of this IFM. By using an EPR couple, partial measurements carried out on one particle out of an EPR pair was found to impose a substantial effect on the other. The result was manifestly non-local effect, and it allowed us to devise a new, Bell-like, inequality. Such a result reinforces the idea that the quantum state represents some "element of physical reality", and not only our knowledge about it.

This technique has further rendered non-locality a continuous process, “going” back-and-forth between the pair of EPR entangled particles. That result is in contrast to the common EPR experiment, in which one measurement totally disentangles the measured particles. Moreover, partial measurement yielded a novel and very genuine method of quantum erasure. Contrary to the known erasure processes, where any traces for the measurement must be erased before any observation can be made, in the proposed method the measurement and the erasure are carried out by macroscopic measurement apparata, in a completely visible way. Last but not least, this technique has proved for the first time that erasure of a measurement can produce non-local effects just like measurement itself.

One of the most evident insights gained from the preoccupation with the conceptual tools, such as interaction-free measurement, partial measurement, and quantum erasure, is the peculiar nature of the quantum particle. As mentioned in chapter 2, quantum particles do not behave like corpuscles, nor do they behave like macroscopic waves. They interact in a way that does not comply with the everyday notions known to us from our macroscopic experience. Hence the debate about “wave or particle” is pointless. The main effort should be invested in learning the way quantum systems behave without trying too hard to equate them with any familiar phenomena from the macroscopic realm.

In Chapter 5 another method of delaying the measurement process was investigated, when several particles in a state of superposition interacted with one another. This way, the particle is not measured directly by a measuring device, but first interacts with a few other particles in a superposition state, such that they all “measure one another,” so to speak, before final measurement of the whole process is carried out.

This way, some interesting results were obtained. For example, when one superposed photon interacted with a row of atoms in a state of superposition, it seemed as if it has interacted with one atom in the middle of the row, while skipping the atoms before and after that one. Such a kind of interaction seems to defy the sequential way the photon should travel in space.

Another result obtained by this method is presented in Chapter 6. A method to entangle two atoms is demonstrated, where the entangling event is carried out in the future, by a single photon that has previously "visited" both atoms, even though they were spacelike separated. An attempt to check Bell's Inequality on an ensemble of such entangled atoms ends up in what seems like an inconsistency of the perceived history of the photon and atoms.

All these investigations lend strong support to the reality of the quantum state, although the local form of realism was refuted time and again. We saw evidence for the real existence of the quantum state in the effect a partial measurement of one EPR particle inflicts on its sibling. Furthermore, we have seen real effects of the wave function in the supposedly *empty* branches of a superposition, as was the non-sequential interaction in chapter 5. More effort is worth taking in this research direction.

I would like to put an emphasis on the physical meaning of the results. A physicist, should she look at the results obtained in chapters 4-6, will immediately state that these results are perfectly plausible and expected, considering the nature of the quantum formalism. However, this view involves years of practice, during which our intuitions are shaped to comply with the weird world of the quantum particles. In order to appreciate the real beauty of the results obtained, one must try and "undo" all these years of training, and to look at the outcomes with a child's eyes.

Chapter 8

Concluding Comments

In the quotation given in the motto at the beginning of this research, Heisenberg stressed the difficulty of communicating the fundamental concepts of quantum mechanics in ordinary language. In the second motto, Wittgenstein asks us to keep silent whereof we can't communicate in ordinary language. Since this dissertation was written in ordinary language, I presume I managed to articulate some of the fundamental concepts. Therefore, in this closing chapter, I take the liberty of saying a few more words on the road ahead. I'll allow myself to indulge in some possible wider implications of the results obtained in the previous chapters.

Quantum mechanics is a branch of physics characterized by many intriguing experiments, which were first gedanken and then turned into real experiments. Today, about a century after quantum mechanics' inception, its ability to produce surprising results is far from exhausting itself. Bell's inequalities, GHZ, quantum erasure, and the peculiar results found in Aharonov's weak measurements, are only a few examples. This work too has shown some novel results that highlight the strangeness of the quantum world.

These experiments also reveal the limitations of the existing interpretations of quantum mechanics. In order to explain the growing bizarreness of quantum effects revealed by modern experiments, quasi-classical interpretations like the Bohmian guide-wave must become more and more surrealistic, while extreme versions of metaphysics like Wheeler's radical instrumentalism become akin to the Strong **Anthropic Principle** [Whe83] or even to **Solipsism**. None of these interpretations is refutable, the exception being collapse theories such as GRW's [GRW86] and GPR's [GPR90] that gives a few falsifiable predictions. My bet, however, is that this theory will be refuted, leaving us with the prevailing, empirically equivalent, interpretations of the theory.

My own belief is that a true change can come only with a new theory of spacetime. There are several reasons to suspect that the prevailing geometric account of time as given by relativity theory does not capture the uniqueness of time in comparison to the other dimensions [ED03]. One prominent such an attribute of time is its apparent asymmetry. Time asymmetry and irreversibility are closely linked to indeterminism. That idea was discussed in two papers I co-authored. In one, [ED99], it was shown that the presence of even a single indeterministic event in a closed system might endow it an intrinsic arrow of time, independent of the system's initial conditions but congruent with the master time arrow of the entire universe, from which the system is supposed to be shielded.

In another, still unpublished paper [DEH01], the relations between irreversibility and indeterminism were surveyed. We have shown that indeterminism involves a modification of the information content of a system. This analysis, when applied to quantum theory, hints that quantum indeterminism, if real, involves either loss of information – as in any collapse interpretation – or, on the contrary, addition of information – as in Aharonov's interpretation [ABL64], where a vector from the future carries information about future measurements results.

Now, does quantum mechanics oblige a true indeterminism (*i.e.*, loss of unitarity)? The jury is still out on this issue, as there is no way yet to disprove the many-worlds interpretation or nonlocal hidden variable theories. But I believe that the above two papers have persuasively shown that, if a decisive proof is ever found for true indeterminism in quantum mechanics, its bearing on time's nature will be far-reaching.

As a result, the peculiar nonlocal nature of quantum measurement, as manifested in experiments like EPR and GHZ (made even more peculiar by the partial measurement presented in chapter 4), together with the time-arrow that follows from any indeterministic physics and the alleged self-contradictory results reached at section 6.7 – all seem to indicate that time's role in physics is much more complex than believed nowadays.

Indeed, this is Aharonov's personal view of his own "two vector" interpretation of quantum mechanics [AR03], which helped him predict some surprising results (as mentioned in page 105). Aharonov (personal communication, 2001) believes that each point in time is visited twice, first by the forward moving wave function and later by the opposite one. Such a use of temporal relations in respect to spacetime itself obviously goes beyond the present relativistic account of spacetime as a "timeless" four-dimensional continuum in which all instances of time have equal reality. My speculation is that the odd phenomena manifested by a system during the time interval between two measurements are the result of two or more "revisions" of that system's history by subtle quantum processes moving back and forth in time. In fact, Penrose's [Pen79, Pen87] theory, which invokes different spacetime manifolds that form during superposition till gravity collapses them, is suggesting something quite similar.

Be these hypotheses as they may, two things are clear. We poorly understand the quantum world and we poorly understand spacetime. It is very probable that the two are intimately related.

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